“Exploiting Multidimensional Design of Experiments and Kriging Methods: An Application to a Satellite Radar System Tradespace and Orbital Transfer Vehicle Tradespace”

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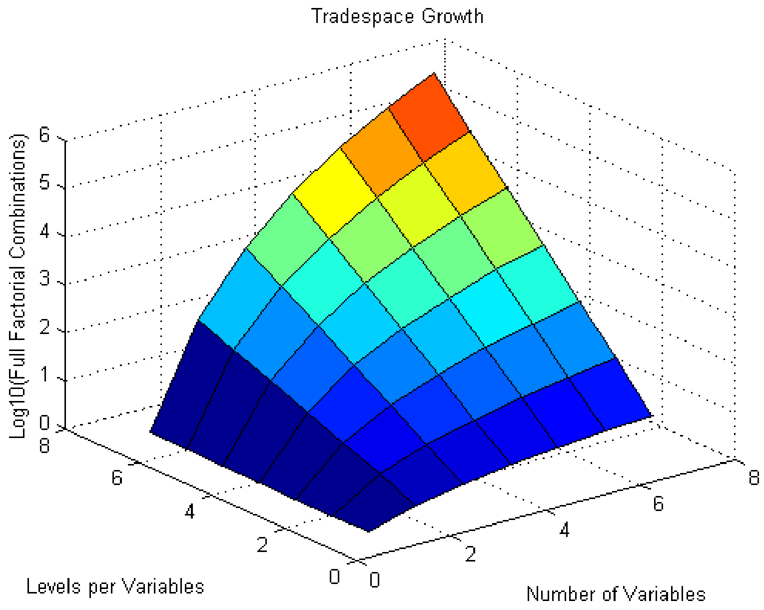
Motivation

- By performing tradespace exploration as early as possible and on as broad of a scale as possible, we avoid premature fixation on potentially non-optimal designs
- Broad scales, however, lead to increased amount of time needed to simulate the tradespace
  - The tradespace grows combinatorically with the number of variables and levels per variable
  - The time to simulate the tradespace is proportional to the execution time for a single design
- By simulating an intelligently sampled subset of the tradespace and interpolating the rest of the points, much time can be saved



Definitions

- “Design Variable”: A variable that defines an aspect of a design.
- “Epoch Variable”: A variable that defines an aspect of a epoch.
- “Valid Range”: The minimum and maximum values allowed for a design or epoch variable based on physical constraints or limitations of the model.
- “Enumerated”: A design level in the set of values spanning a valid range.
- “Design-Epoch Pair”: A combination of a single design and single epoch
- “Simulated”: A design-epoch pairing whose attributes are calculated via a performance model.
- “Unsimulated”: A design-epoch pairing whose attributes will be generated by means other than a performance model.

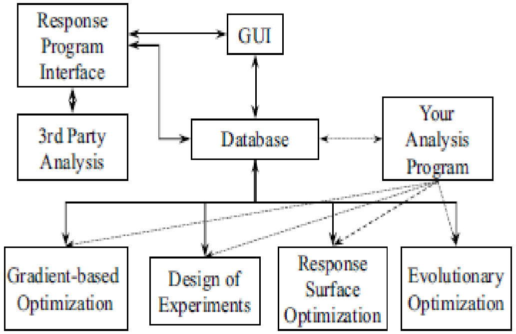
Existing Methods

- Architecture Enumeration and Evaluation (AEE) process developed by UTRC (Murray et al 2011)
  - Builds the architecture piece by piece, gradually building up to a full architecture and eliminating entire subtrees of invalid architectures
  - Each architecture is “simulated” without necessarily needing to be examined
- Others calls for varying model fidelity (Howell et al 2005)
  - Use lower fidelity models to evaluated full tradespace
  - Use higher fidelity models to evaluate designs of interest, as identified by lower fidelity model
- System Platform for Integrated Design in Real-Time (SPIDR) (Kichkaylo et al 2012)
  - Uses artificial intelligence-based search and an optimization engine
  - “Constraint-based design synthesis engine”
- Applied Research Laboratory (ARL) Tradespace Visualizer (ATSV) (Stump et al 2009)
  - Given a simulated tradespace, allows users to select areas of interest for a response surface model to populate, guided by a differential evolution algorithm
  - Requires measure of fitness

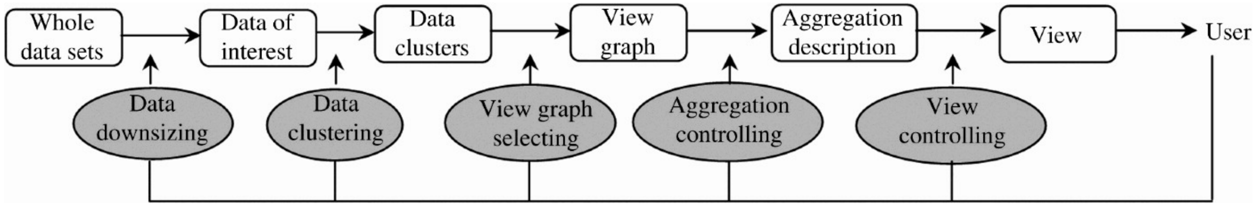
Performance, rather than binary validity or optimization, is the goal for this study. We will only consider the case when one model fidelity is available.

Existing Methods Cont'd

- Previous explorations of Kriging in engineering and design optimization (Simpson et al. 2001, Martin et al. 2004, Kleijnen 2009)
  - The value of Kriging in place of response surface models has been previously noted
  - Shortcomings of Kriging also addressed
    - Lack of guidance on what type of Kriging to use
    - Lack of readily available software for implementation
- VisualDOC (Balabanov et al 2002)
  - Commercially available tool
  - Given a database (simulated tradespace), allows your analysis program to take advantage of many optimization and DOE tools
  - Very modular, a property which ETAM seeks to mimic



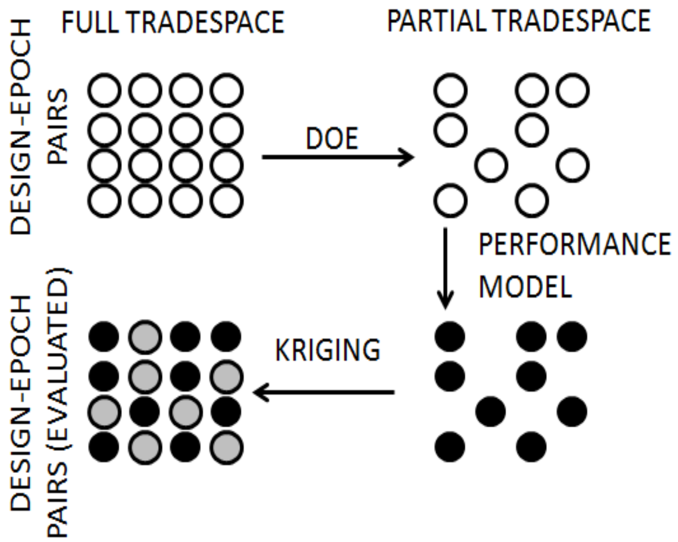
- Interactive multiscale-nested clustering and aggregation (iMSNCA) framework (Zhang et al. 2011)
  - User in the loop
  - Uses graphs and data downsizing to minimize time required to characterize a design tradespace



Overview of ETAM*

3 steps, leveraging DOE and Kriging

1. Isolate appropriate data sets from the fully enumerated tradespace
   - "Appropriate" meaning non-Krigable variables are held constant
2. Intelligently sample the isolated data sets to form a training set
3. Interpolate the remaining points in each isolated set using the training set



*ETAM: Expedited Tradespace Approximation Method

Step 1: Isolating Appropriate Data Sets

- Each design and epoch variable must be classified

Variable Type	Explanation	Example	Classification
Ratio	The difference between two values is meaningful and there is a definition of zero.	Temperature (K or R), Power, Length, Time	Krigable
Interval	The difference between two values is meaningful.	Temperature (F or C),	Krigable
Ordinal	Order matters but not the difference between values.	Technology Readiness Level (TRL)	Non-Krigable
Nominal	Mutually exclusive, but not necessarily ordered (i.e. categorical).	Binary Variables (On/Off), ID #s	Non-Krigable

- Since the difference between independent variables is considered in Kriging, only interval and ratio variables are considered Krigable
- “Appropriate data sets” have constant values for each non-Krigable variable and vary along the Krigable Variables and will heretofore be referred to as “Krigable subsets”

Step 2: Selecting the Training Set

- Each Krigable Subset has the same Krigable variable combinations
  - The combinations that will be simulated are referred to as the “training set”
  - The training set is the same for each Krigable subset
- The size of and selection of the training set is a function of the type of design of experiments (DOE) being used
  - Smaller training set means more time savings
  - Smaller training set can also mean less accurate interpolated data

Step 2 is where the tradeoff between time savings and accuracy of data originates

Step 3: Interpolating the Remainder of the Tradespace

- The method for interpolation used for step 3 was Kriging (Papalambros 2000, Chiles and Delfiner 1999)
  - Kriging is a method taken from geostatistics used to estimate the value of a random variable based on the values of that variable at surrounding locations (Papalambros 2000)
    - Treats error as a functional departure from a polynomial approximation function
    - "Ordinary Kriging" is the specific type being used, which assumes the random variable has a constant, albeit unknown, mean
  - Other methods of interpolation could be used as well

Step 3 Cont'd

- A Kriging vector is generated for each attribute using the training set variables, attribute values, and a variogram
  - Lambdas are the vector elements
  - *A* is the training set size

$$\begin{cases} \sum_{\beta} \lambda_{\beta} \sigma_{\alpha\beta} = 2\sigma_{\alpha 0} & \alpha = 1, \ldots, A \\ \sum_{\alpha} \lambda_{\alpha} = 1 \end{cases}$$

- The variogram is a measure of spatial dependence
  - Power law variogram is used for its simplicity
  - *r* is the cartesian distance between the two dependent variable values
  - Beta is fixed and alpha is determined by the cartesian distances between all points in the training set
  - Represented as sigma in Kriging vector equations

$$v(r) = \alpha \cdot r^{\beta}$$

Step 3 Cont'd

- The points in the Kriging set (difference between Krigable subset and training set) are then interpolated using the Kriging vector

$$y_0^* = [\mathbf{D}* \quad 1] * \mathbf{K}$$

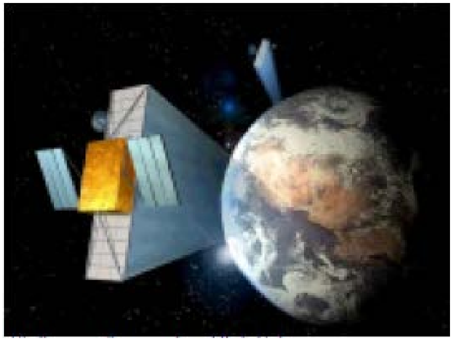
  - y_0 is the Kriged attribute
  - **D** is the vector of variogram values between the training set points and the point being Kriged
  - **K** is the Kriging vector
- This is done for every point in the Kriging set, for each attribute, for each Krigable subset

Matrix operations like this in MATLAB can potentially be orders of magnitude faster than a performance model

APPLYING ETAM TO SATELLITE RADAR

Satellite Radar Tradespace

- Satellite constellation for 24-hour, all-weather radar-based imaging and tracking
- Nontrivial data set: 23,328 designs across 972 epochs
  - 8 Design Variables
  - 6 Epoch Variables
  - 12 Attributes



- Purpose of the case study is to validate the accuracy of ETAM against an existing fully simulated tradespace

Characterizing the Variables

Design Variable	Scale Type	Valid Range	Enumerated Levels	# Levels
Altitude	**Ratio**	**800 – 1500 [km]**	**800, 1200, 1500 [km]**	**3**
Constellation Configuration	Nominal	1 – 8 [int]	8 walker IDs	8
Antenna Area	**Ratio**	**10 – 100 [m^2]**	**10, 40, 100 [m^2]**	**3**
Peak Transmit Power	**Ratio**	**1.5 – 20 [kW]**	**1.5, 10, 20 [kW]**	**3**
Radar Bandwidth	**Ratio**	**0.5 – 2 [GHz]**	**0.5, 1, 2 [GHz]**	**3**
Communication Downlink	Nominal	Relay or Direct Downlink	Relay, Direct Downlink	2
Tactical Communication	Nominal	Able or Not Able	Able, Not Able	2
Maneuver Capability	Ordinal	1 – 4 [x base fuel]	1x, 2x, 4x Base Fuel	3

Characterizing the Variables Cont'd

Epoch Variable	Scale Type	Valid Range	Enumerated Levels	# Levels
Available Radar Technology	Ordinal	9 – 3 [TRL]	Mature, Medium, Advanced	3
Communications Infrastructure	Nominal	0 – 2 [int]	AFSCN, WGS + AFSCN, Third	3
Target Set	Nominal	1 – 60 [int]	Lookup Table of 9 Regions and Ops Plans	9
Collaborative AISR Assets	Nominal	Available or Not Available	Available, Not Available	2
Threat Environment	Nominal	No Jamming or Hostile Jamming	No Jamming, Hostile Jamming	2
Mission Priorities	Nominal	SAR <GMTI, SAR=GMTI, SAR<GMTI	SAR < GMTI, SAR = GMTI, SAR < GMTI	3

Isolating the Krigable Subset

- All variables except the 4 Krigable variables are held constant
  - Altitude (3 enumerated levels)
  - Antenna Area (3 enumerated levels)
  - Peak Power (3 enumerated levels)
  - Bandwidth (3 enumerated levels)
  - Total of 81 design points in each Krigable Subset

Selecting the Training Set

- Box-Benkhen was selected as the DOE method of choice (Box and Benkhen 1960)
  - Especially convenient for Satellite Radar because it considers three levels per variable
  - Only valid for >2 variables
    - 8 of the 12 attributes are a function of 1 or 2 variables, so they will not be considered
    - Number of boxes, tracking latency, imaging latency, and targets per pass were functions of 3 variables, hence a 13 point training set was used
- Used a binary domain mapping matrix to select which points to include in the 13 (Danilovic and Browning 2007)

ETAM Results

- Overall results

Attribute	Mean (Training Set Points)	E[|Difference|]	Std[|Difference|]	Average Error
Num Boxes	12.674	2.574	6.666	n/a
Track Latency	52.545	2.935	7.599	2.81%
Image Latency	393.059	9.386	16.218	2.01%
Targets per Pass	23676.942	2548.697	9748.730	31.51%

- Kriging was "100% successful" for the other 8 attributes since the 13 training points were selected such that an exact match for every point in the Kriging set was contained

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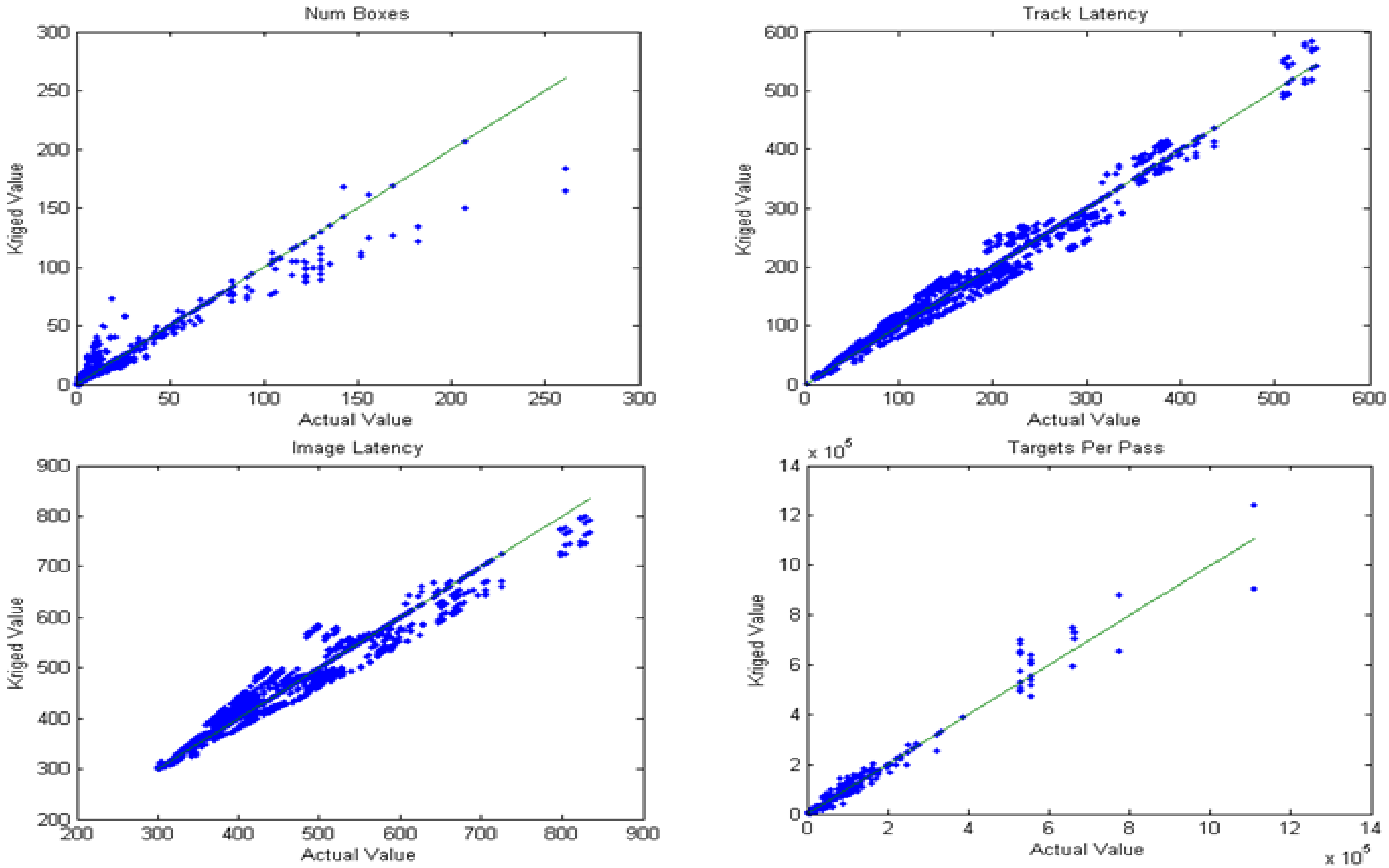
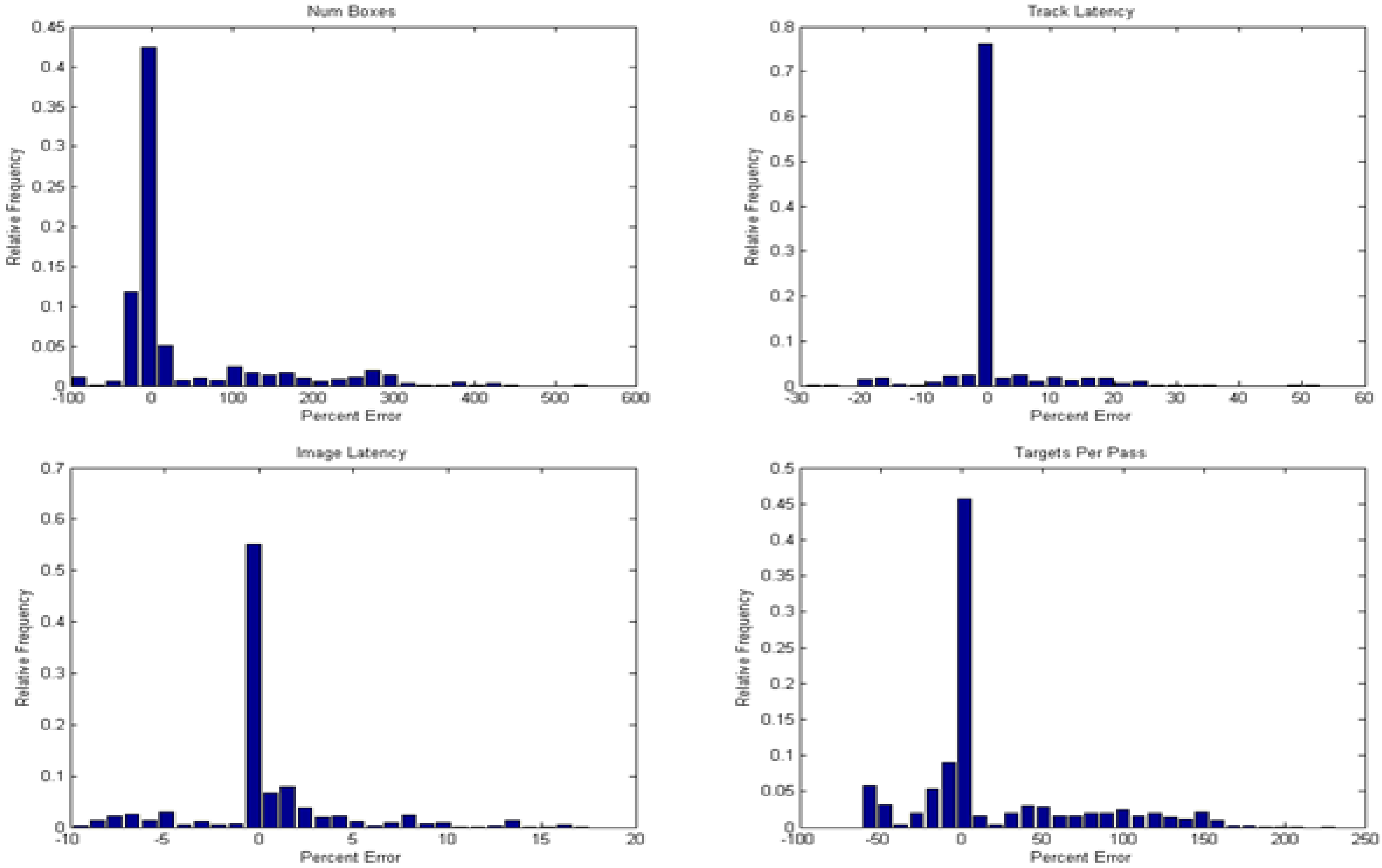
ETAM Results Cont'd



ETAM Results Cont'd



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Second Case: Space Tug

General purpose orbit transfer vehicle

3 design variables → 128 designs

- Prop type (biprop, cryo, electric, nuclear)
- Fuel mass
- Capability level

8 prefs x 2 contexts → 16 epochs

8 preference sets (of 4 attributes)

- Delta-V potential
- Mass able to be manipulated
- Speed
- Cost

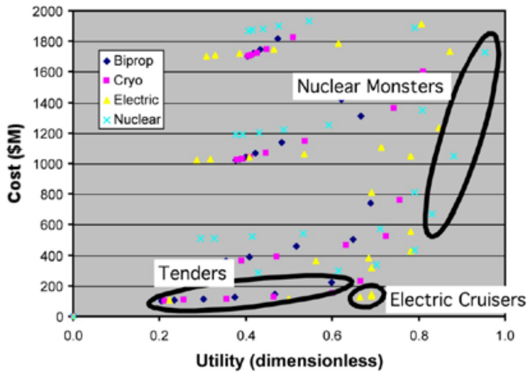
2 contexts

- Present vs. future technology level

Design Space

- Manipulator Mass
  - Low (300kg)
  - Medium (1000kg)
  - High (3000 kg)
  - Extreme (5000 kg)
- Propulsion Type
  - Storable bi-prop
  - Cryogenic bi-prop
  - Electric (NSTAR)
  - Nuclear Thermal
- Fuel Load - 8 levels

McManus and Schuman, 2003



- Simple performance model
  - Delta-V calculated from rocket equation
  - Binary response time (electric propulsion slow)
  - Capability solely dependent on manipulator mass
  - Cost calculated from vehicle wet and dry mass

McManus, H., and Schuman, T., “Understanding the Orbital Transfer Vehicle Tradespace,” AIAA-2003-6370, AIAA Space 2003, Sept. 2003.

ETAM Discussion

- Applied to Space Tug as well

Attribute	Mean (Truth Data)	E[|Difference|]	Std[|Difference|]	Average Error	Spearman Rho
Cost	$976,640,000	$3,722,500	$5,368,500	0.55%	0.9999
Delta V	5.2437 km/s	0.898 km/s	1.6964 km/s	0.29%	0.9617

- In both cases, the results were very promising
  - Kriging has difficulty with attributes that have values spanning many orders of magnitude
- Requires weighing potential losses in accuracy to the amount of tradespace that can be explored in feasible amount of time

ETAM Discussion

- ETAM Time Dilation Factor

$$TDF = \left(\frac{E + k \cdot n \cdot t}{n \cdot t} \right) \cdot 100$$

- *n*: # design-epoch pairs in fully enumerated tradespace
- *t*: execution time of performance model for a single design-epoch pair
- *k*: ratio of training set to Krigable subset
- *E*: upfront time to integrate tradespace into ETAM coding framework

Case Study	*E*	*k*	*n*	*t*	Time Dilation Factor
Satellite Radar	8 hours	13/81	6,718,464	≈26s	16.07%
Space Tug	2 hours	32/96	384	0.0015s	1,250,000%

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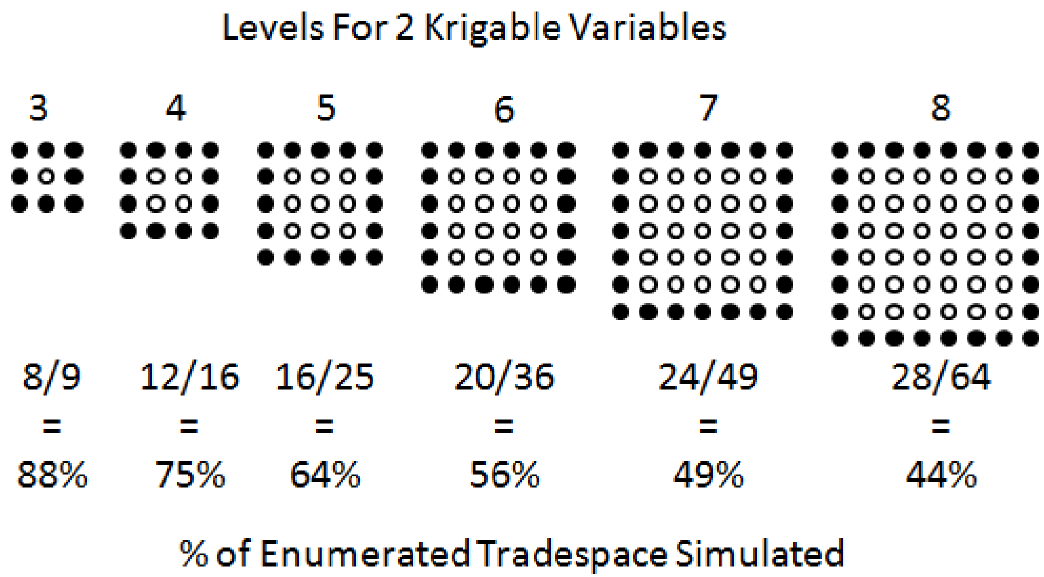
ETAM Limitations

- Savings limited by variable composition of the tradespace in question
- Accuracy on the “edges” of the tradespace is diminished by the nature of the Kriging estimate
  - Can be addressed by including all points on the surface of the n-dimensional hyper-cube
  - Savings increase with the number of levels



Conclusions

- The validity of ETAM as a method for approximating large tradespaces was demonstrated through application to two case studies
- Both savings (time) and costs (accuracy) are dependent on the makeup of the variables in the tradespace being examined

QUESTIONS?