Using Pareto Trace to Determine System Passive Value Robustness

Adam M. Ross, Donna H. Rhodes, and Daniel E. Hastings

Engineering Systems Division, Massachusetts Institute of Technology
Building E38-576, 77 Massachusetts Avenue
Cambridge, MA 02139 USA

Abstract—**An important role of system designers is to effectively explore the tradespace of alternatives when making design decisions during concept phase. As systems become more complex, formal methods to enable good design decisions are essential; this can be empowered through a tradespace exploration paradigm. This paper demonstrates the use of the Pareto Trace and associated metrics to identify system alternatives across tradespaces with high degrees of passive value robustness—alternatives that continue to deliver value to stakeholders in spite of changes in needs (attributes) or context. A value-driven tradespace approach is used to represent the baseline performance versus cost of a large number of system alternatives. The classical notion of Pareto Set is extended to identify alternatives and their characteristics that lead to their inclusion in Pareto Sets across changing contexts. Using a low-earth orbiting satellite case example, five types of context changes are used to demonstrate this method: 1) addition or subtraction of attributes; 2) change in the priorities of attributes; 3) change in single attribute utility function shapes; 4) change in multi-attribute utility aggregation function; and 5) addition of new decision maker. This approach demonstrates the ability for system designers to pose questions about assessment of alternatives during early conceptual design. Suggestions for application of Pareto Trace beyond the case example are discussed and presented, including application of a "fuzziness" factor and statistical measures. In particular, distinctions from traditional sensitivity analysis are made, as well as linkages to dynamic analysis for discovery of generalized value robust alternatives.**

Keywords-system design; value robustness; pareto set; pareto trace; changeability metrics; tradespace exploration

I. INTRODUCTION

An important role of the system designer is to effectively explore the tradespace of alternatives when making design decisions during concept phase. As systems become more complex and integrated, formal methods to enable good design decisions are essential and necessitate a shift from simple decision approaches to a tradespace exploration (TSE) paradigm [1].

TSE methods [1], supported by parametric modeling and value based approaches [2,3], provide the capability to effectively explore alternatives, beyond simple trade-off analysis [4], giving the designer an ability to incorporate options and issues raised during the analysis, and the means to compare many system alternatives. Over time, system decision makers may change their mind on which system attributes provide value and the system that is *value robust* will display attributes to match the new expectations. In some cases, a change to the system may be necessary, but in others, a physical system change may not be required, particularly if the system contains latent value [5]. The achievement of value robustness [6] can be accomplished through either passive or active means [7]. Active value robustness can be achieved through a strategy of pursuing designs with increased changeability and accessibility to likely high value regions of a tradespace. The subject of this paper is a metric for evaluating passive value robustness in tradespaces; this type of value robustness can be achieved by developing "clever" systems, which may have excess capability or a large set of latent value, increasing the likelihood of being able to match new value expectations without requiring a system change.

One of the goals of a system designer is to maximize value delivery at efficient levels of resource expenditure. A classical concept for evaluating efficiency, used in economics, as well as multi-objective optimization, is *Pareto Optimality*. Pareto Optimality is achieved when resources can no longer be distributed to improve at least one individual without making others worse off than before [8]. Extending this concept to multi-objective decision making, Pareto Optimality is achieved when a solution is non-dominated, that is, a solution cannot be improved in a particular objective score without making other objective scores worse. Multi-Attribute Tradespace Exploration (MATE) is a method that evaluates a large number of alternatives in terms of cost, and utility perceived by a decision maker [4, 6]. Fig. 1 illustrates an example Utility-Cost tradespace. Maximizing the utilities and minimizing the costs are the objectives for the concept selection problem addressed in this paper.

---

[1] The specific TSE method used in this research is Multi-Attribute Tradespace Exploration (MATE), a formal framework for tradespace exploration during system design, which uses multi-attribute value-driven design (e.g. multi-attribute utility theory) coupled with tradespace exploration [4,6].
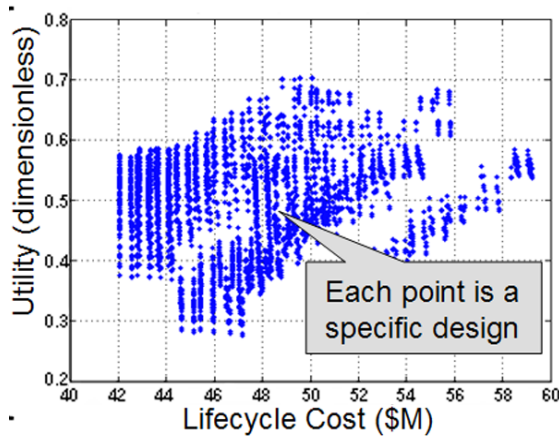
Figure 1. Example Utility-Cost tradespace for evaluating large number of design alternatives [5].

Compounding classical optimization problem for determining the "best" design alternative (i.e. Pareto Optimal solutions in utility-cost space), is the fact that the expectations for, as well as the context of the systems change over time, resulting in time-varying utility and cost scores for the various alternatives under consideration [9]. In order to quantitatively identify design alternatives that "do well" across these changes in expectations and contexts, the concept of Pareto Optimality is extended across temporally distinct tradespaces. The proposed metric is called the *Pareto Trace*.

II. PARETO TRACE DEFINED

Pareto Trace of a design alternative is defined as "the number of Pareto Sets containing that design," where a distinct Pareto Set is calculated for a particular utility-cost tradespace with a fixed set of expectations and context (i.e. constraints, objective functions, and decision variables) [5, 7]. Calculating the Pareto Trace of a design, or *Pareto Tracing*, provides a mechanism for the analyst to discover and quantify designs that are most passively value robust. A design with a high Pareto Trace, as shown in Fig. 2 below, is one that appears in many Pareto Sets across various Epochs, or a period of time with fixed expectations and context. These designs can be used as "attractors" for future scenarios, as a goal state for change pathways [9]. Additionally, analyzing the design characteristics
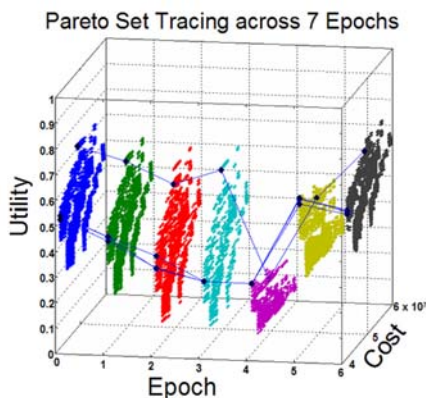

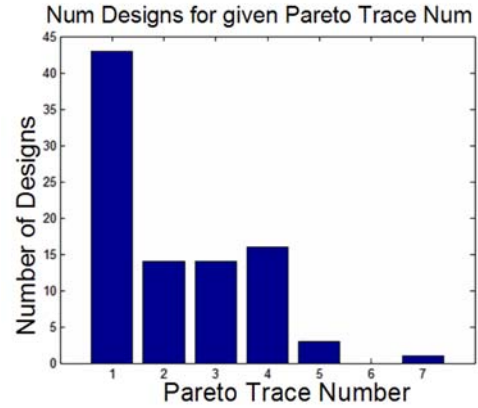Figure 2. Pareto Trace illustrated across 7 epochs [5].

Figure 3. Example distribution of Pareto Trace across 7 epochs (N=9930) [5].

of alternatives with a high Pareto Trace gives system designers insight into the most value-delivering and efficient combination of design parameters in the face of changing contexts. Statistics such as in Fig. 3, illustrate the distribution of Pareto Trace numbers across a tradespace, giving an indication of the frequency of passively value robust designs.

This paper demonstrates the use of the Pareto Trace to identify system alternatives with high degrees of passive value robustness, meaning alternatives that continue to deliver high value to stakeholders in spite of changes in needs or context. A value-driven tradespace approach is used to represent the baseline performance versus cost of a large number of system alternatives.

In addition to calculating a Pareto Trace, one may be inclined to recognize that uncertainty, especially in the conceptual design phase, may reduce confidence in the assessed objective function values (i.e. utilities and costs) in the tradespace. One approach to incorporating this uncertainty is to add "fuzziness" to the Pareto set. *Fuzzy Pareto Optimality* is introduced by [10], and extends the classical notion of Pareto Optimality to including those solutions within K fraction of the Pareto Frontier (the non-dominated solution space). Formally, [10] defines Fuzzy Pareto Optimality for the goal of minimizing objectives J^1 and J^2 as:

J^1 dominates J^2 if:

$$J^1 + K(J^{max} - J^{min}) \leq J^2, \text{ and } J^1 \neq J^2 \tag{1}$$

$$J^1_i + K(J^{max}_i - J^{min}_i) \leq J^2_i \ \forall \ i \text{ and} \tag{2}$$

$$J^1_i + K(J^{max}_i - J^{min}_i) < J^2_i \text{ for at least one } i, \tag{3}$$

where K represents a user definable value between 0 and 1. According to [10], for values of K "between 0 and 1, the Fuzzy Pareto Optimal set will include all solutions that are within the $K(J^{max}_i - J^{min}_i)$ rectangle offset from the (weak) Pareto frontier."

Analogous to the Pareto Trace, the *Fuzzy Pareto Trace* of a design alternative is defined as "the number of Fuzzy Pareto Sets containing that design." The Fuzzy Pareto Trace will clearly vary based on the value assigned to K, however, in the spirit of tradespace exploration, variation in the K value will give insight into the impact of uncertainty on various design alternatives and their apparent passive value robustness.

One may notice that the Pareto Trace for a design is dependent on the particular epochs considered in the analysis. In order to reduce this bias, one final variation on the Pareto Trace is now defined: the *Normalized Pareto Trace* (and the analogous *Normalized Fuzzy Pareto Trace*). Since the number of possible variations in context and expectations is infinite, the absolute score of a Pareto Trace for a design is unbounded.. In order to account for this, the Pareto Trace can be normalized by the number of epochs considered. Additionally, designs may achieve high Pareto Trace due to selection bias in the epochs considered. Stabilization of the Normalized Pareto Trace across various numbers and types of evaluated epochs gives confidence that the score is not overly biased by the enumeration and sampling of possible epochs.

Two case studies are now presented to illustrate the use of Pareto Trace and Fuzzy Pareto Trace for identifying passively value robust design alternatives from within a tradespace.

III. PARETO TRACE: CASE STUDY APPLICATION

The classical notion of Pareto Set is extended to identify alternatives and their characteristics that lead to their inclusion in Pareto Sets across changing contexts. Using a low-earth orbiting satellite case example, five types of expectation changes are used to demonstrate the usefulness of this method: 1) addition or subtraction of attributes; 2) change in the priorities of attributes; 3) change in single attribute utility function shapes; 4) change in multi-attribute utility aggregation function; and 5) addition of new decision maker. In total, 60 tradespaces are generated through these changes in expectations. The tradespaces in this case example were generated using parametric and simulation models of a low-earth orbiting satellite system to evaluate thousands of design alternatives in terms of cost, attributes, and utilities. The attributes are the system characteristics chosen by a decision maker as relevant decision metrics for distinguishing between alternatives. Reference [4] describes the case study, X-TOS, at length, including the design parameters, preferences, and context for the system. The epochs considered in the analysis below were evaluated and described at length in [7].

The Pareto Sets are determined through non-dominated solutions in terms of cost and multi-attribute utility of alternatives. Multi-attribute utility is calculated as follows:

$$KU(\underline{X})+1=\prod_{i=1}^{M}\left[Kk_{i}U^{i}(X^{i})+1\right], \qquad (4)$$

where $K+1=\prod_{i=1}^{M}\left[Kk_{i}+1\right]$ [3]. $\qquad (5)$

The notation is unfortunate since the K in (4) and (5) is not the same K in (1-3). In this equation, U($\underline{X}$) is the multi-attribute utility score of an alternative, aggregating the single attribute functions, U^i, of attribute X^i, where i varies from 1 to the number of attributes in the set of a decision maker. The k_i varies from 0 to 1, representing the priority of attribute X^i, and is elicited from a decision maker. A multi-attribute utility score is calculated per decision maker. In this example, a single decision maker is assumed for simplicity, unless otherwise stated.

A. Changing attributes (Exp1a)

The first change considered is change in attribute set. The attribute set is the set of performance objectives defined by the system decision maker. For this system, the attribute set includes the following characteristics of the system: Data Lifespan, Latitude Diversity, Equator Time, Latency, and Sample Altitude (or {DL, LD, ET, L, SA} as shorthand). Changes to this set included dropping one or more of these attributes from the set, as well as addition of satellite mass. The baseline k_i vector is [0.3, 0.125, 0.175, 0.1, 0.425]. For this variation in epoch, seven epochs were generated: one with baseline expectations and five new attribute sets formed by dropping the least important remaining attribute from the prior set (i.e. first dropping latency, second dropping latitude diversity, etc.) The last epoch was generated by adding satellite mass as a new attribute, since it is often used by technical analysts as a proxy measure for "waste." Fig. 4 below illustrates the results of Pareto Tracing across these seven epochs, with four designs (out of 9930 alternatives) having Pareto Traces of five (designs 903, 1687, and 2535) or seven (design 2471).

B. Changing priorities (Exp1b)

The second change case considered is a change in attribute priorities. The baseline value of the k vector is given above and is varied one at a time and two at a time to result in 32 epochs. First, the most important attribute priority, sample altitude, is varied over the range 0.1 to 0.9 in steps of 0.1 to generate 9 epochs. Next, the priority of the second most important attribute, data lifespan, is varied over the range 0.1 to 0.9 in steps of 0.1 to generate 9 more epochs. Lastly both sample altitude and data lifespan priorities are varied together randomly in steps of 0.1 to generate 14 more epochs. Eight designs had Pareto Trace of 32 for this set of epochs, including designs 903, 951, 952, 967, 1687, 2471, 2519, and 1535.
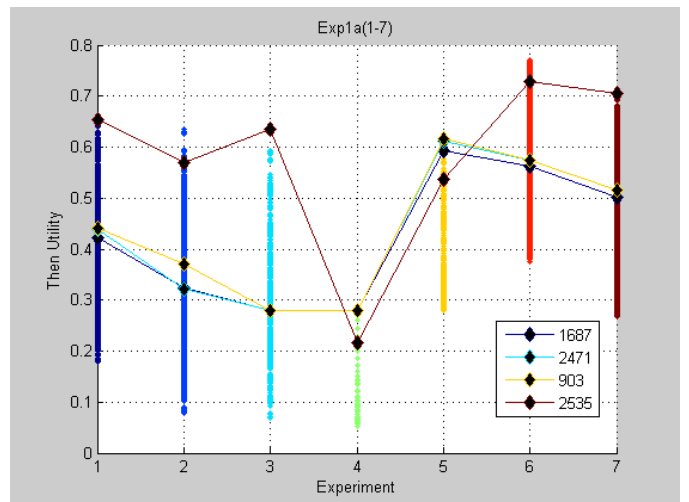

Figure 4. Pareto Tracing of changing attributes epochs, showing top four design numbers.

$$\text{MAUF} \quad U = \frac{\left(\prod_{i=1}^{N}\left[Kk_iU_i+1\right]\right)-1}{K}$$

$$\text{M1} \quad U = \prod_{i=1}^{N} k_iU_i \qquad \text{M2} \quad U = \prod_{i=1}^{N}\left[k_iU_i+1\right]$$

$$\text{M3} \quad U = \sum_{i=1}^{N} k_iU_i \qquad \text{M4} \quad U = \prod_{i=1}^{N}\left[Kk_iU_i+1\right]$$

Figure 5. Various aggregation functions considered.

C. Changing single attribute utility functions (Exp1c)

The third change case considered a change in the single attribute evaluative function form. Typically the single attribute utility functions are elicited from a decision maker through formal interviews, but errors in elicitation or lack of time may prevent confidence or availability of accurate functions [3, 4, 7, 11]. Variation in the functional form from the non-linear elicited version to a simpler linear form was conducted for this set to generate a set of 16 epochs. Fifteen designs had Pareto Trace of 16 for this set of epochs, including designs 903, 951, 967, 1687, 1735, 2487, 2503, 2519, 2535, 6891, 6915, 6939, and 6963.

D. Changing aggregation function (Exp 1d)

The fourth change case considered a change in the aggregation function for the multi-attribute utility score. Various methods in the literature and practice point to linear weighted sums, aggregation of attributes (instead of utilities), inverse multiplicative functions, and various other schemes for aggregating the goodness to rank alternatives. In this set of epochs, five different aggregation functions were used to assess the multi-attribute utility score, shown in Fig. 5, which resulted in five more epochs considered. Twenty-six designs had Pareto Trace of 5, meaning they were insensitive to these radical changes in aggregation function. Among these designs were designs 2471, 903 and 1687, but not design 2535.
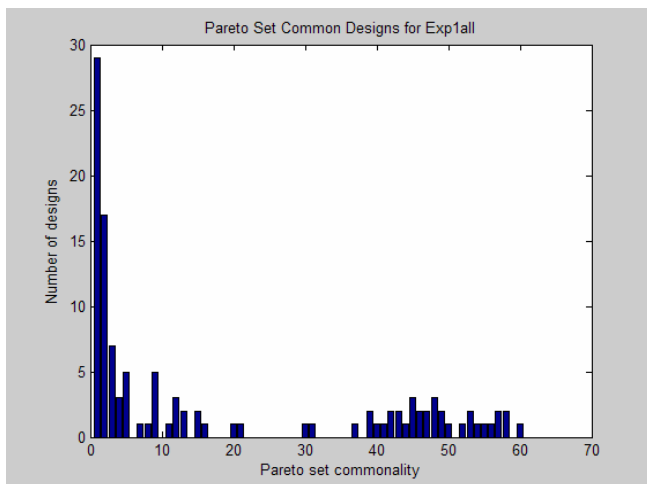

Figure 6. Distribution of Pareto Trace across all 60 epochs showing very few designs with high Pareto Trace.
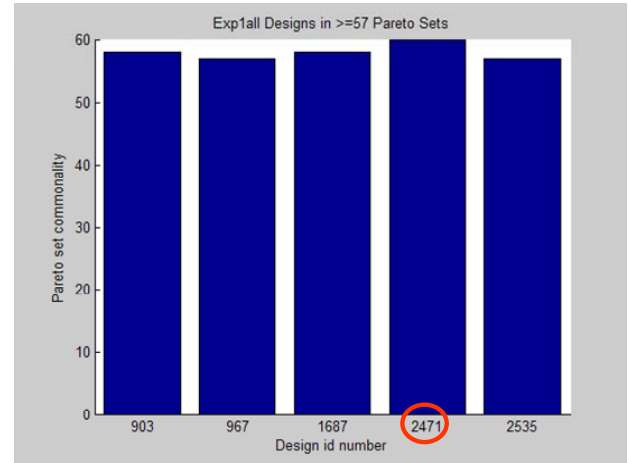

Figure 7. Highest Pareto Trace results across all epochs by design id number.

E. Changing decision maker set (Exp2a)

As a final change case considered, a second decision maker was introduced with distinct expectations. The attributes for the second decision maker were: Cost to IOC, Development Time, Satellite Lifetime, and Satellite Mass, with k_i priorities of [0.4, 0.3, 0.2, 0.2]. Pareto Sets were calculated for Utility-Cost tradespaces per decision maker, as well as a Pareto Set for $\text{Utility}^{decisionmaker1}$-$\text{Utility}^{decisionmaker2}$-Cost. Pareto Tracing across the two decision maker Pareto Sets results in six designs that appear in both, including designs 2471, 2487, 6867, 6891, 6915, and 6959. Interestingly, the joint Pareto Set (U^{DM1}-U^{DM2}-Cost) contains 122 designs, while Pareto Set (U^{DM1}-Cost) contains 36 and Pareto Set (U^{DM2}-Cost) contains 26 designs. The "extra" designs in the joint Pareto Set (122-36-26+6=66 designs) can be called "compromise" solutions and exist as "efficient" tradeoffs between the three objectives, making no assumption about the relative importance of the two decision makers. This type of analysis can be used to identify solutions that can form the basis for negotiations (i.e. "compromise" solutions) or clear win-win (i.e. the six Pareto Trace two solutions that are "best" for both decision makers).

F. Case Study Summary

Across these 60 tradespaces, one design (2471) is identified as being in all 60 Pareto Sets, thereby suggesting the design has high passive value robustness. Several other designs are identified to have a Pareto Trace number of 57 out of 60. Investigation of these alternatives in Fig. 8 reveals that they all share a common spacecraft design, but differ in orbit characteristics.

While more illustrative than rigorous, the case application demonstrates the ability for analysts and decision makers to pose the following questions about assessment of alternatives during early conceptual design:

• What if you don't elicit the "right" attribute priorities? (Exp1a)
• What if you don't elicit all of the "right" attributes? (Exp1b)

DV	2471	903	1687	2535
Inclination	90	30	70	90
Apogee	460	460	460	460
Perigee	150	150	150	290
Com Arch	TDRSS			
Delta V	1200			
Prop Type	Chem			
Pwr Type	Fuel Cell			
Ant Gain	Low			

Figure 8. Design variable values for high Pareto Trace designs identified in case study.

- What if you don't elicit the "right" utility curve shape? (Exp 1c)
- What if you don't use the "right" utility aggregating function? (Exp1d)
- What if a second decision maker enters the mix? (Exp2a)

IV.   FUZZY PARETO TRACE: CASE STUDY APPLICATION

A second case application was applied to a satellite radar system in order to demonstrate the use of Normalized Fuzzy Pareto Trace to identify value robust designs. The case application is described in [12] and [13]. For this case study, 23,328 designs were evaluated by parametric models and simulations in terms of 12 attributes segregated in two utility functions (imaging mission and tracking mission). Unlike the prior case study, whose epochs represented changes in expectations alone, the epochs in this case study were generated through a parameterization of both context and expectations. Enumeration of the epoch variables, including factors such as technology, national security policy, and infrastructure availability, resulted in 648 distinct epochs [12].

Of these, a random sampling was conducted to downselect to 145 epochs. An additional 100 epochs were sampled from the set to test the stability of the Normalized Pareto Trace numbers, resulting in a total of 245 epochs evaluated. Fig. 9 illustrates the Normalized Pareto Trace across the 145 and 245 epochs. These Normalized Pareto Traces (NPT) were calculated across four different Pareto Sets: Utility^{image}-Cost, Utility^{track}-Cost, Utility^{image}-Utility^{track}, and Utility^{image}-Utility^{track}-Cost. While the NPT values did change a little across the increased sampling, the set of designs identified as scoring high did not change, leading to confidence in the sampling of the epoch space. Design 3435 was found to have the highest Pareto Trace at both sampling levels, suggesting it to be most passively value robust for these changing epochs [13].

Using high Normalized Pareto Trace scores as a filter, the joint and compromise Pareto Sets were calculated across the two mission areas, similar to the two decision maker case above. This resulted in the identification of two designs in the Pareto Set of both missions (across a large number of epochs), as well as more than 23 "compromise" designs with high NPT that would not be considered "optimal" for a particular mission, but represents an efficient tradeoff between the two missions across a large number of changing contexts and expectations.

The next step in the NPT analysis for this case study is the introduction of the fuzziness factor K to add tolerance to uncertainty, not throwing out designs that are "close" to Pareto Optimal. K is varied from 0 to 0.15 in steps of 0.05, where K=0 is equivalent to no fuzziness. Fig. 10 illustrates the distribution of Fuzzy Normalized Pareto Trace across the 245 epochs. As expected, as the fuzziness factor increases, so too does the number of high NPT designs. Interestingly at a fuzziness factor of only 0.05, two designs have an NPT of 1, meaning they appear in all 245 fuzzy Pareto Sets. As uncertainty increases, the number of designs in all fuzzy Pareto Sets increases, as shown in Table 1. Inspecting the design numbers that have highest NPT, however, reveals that the one highest NPT for
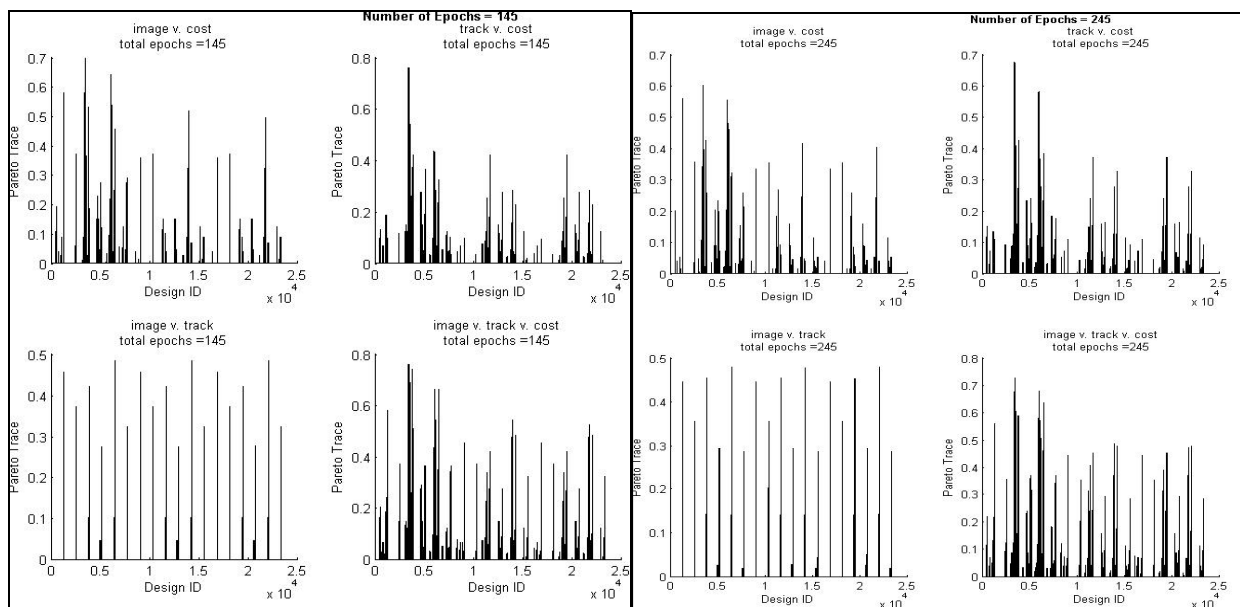

Figure 9. Normalized Pareto Trace across 145 epochs (left) and 245 epochs (right) versus design ID
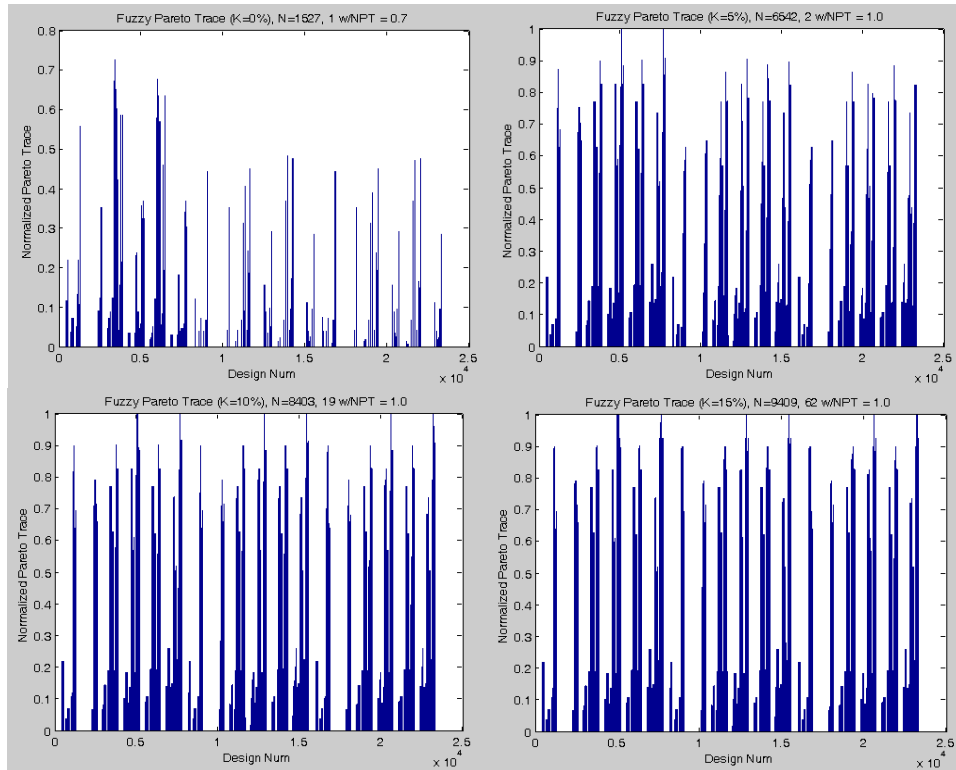
Figure 10. Fuzzy Normalized Pareto Trace across 245 epochs, with fuzziness K varying from 0, 0.05, 0.10, and 0.15.

K=0 is not the same design that appears as highest NPT for K=0.05, 0.10, or 0.15. In fact, design 3435, which is the highest NPT for no uncertainty, does not appear in any of the sets with highest NPT for K>0. Instead, the first two designs to appear at K=0.05 are designs 5067 and 7659, which join a growing set of highest NPT as K increases. Further work is ongoing to investigate the impact of fuzzy NPT insights and stability of passive value robust designs across more epochs.

TABLE I.   FUZZY NORMALIZED PARETO SETS WITH HIGHEST NPT

K	Num Designs with NPT>0[a]	Highest NPT	Num Designs with highest NPT
0	1527	0.7	1
0.05	6542	1.0	2
0.10	8403	1.0	19
0.15	9409	1.0	62

a. Total number of designs is 23,328

V.   DISCUSSION

The concepts of Pareto Trace, Fuzzy Pareto Trace, and Normalized Pareto Trace were introduced and illustrated in order to demonstrate a quantitative approach for identifying system designs that are cost-benefit efficient across changing contexts and expectations.   While traditional sensitivity analysis and multi-objective optimization may result in similar mathematical treatment (e.g. varying objective function and constraint weights), the underlying conceptual frame for Pareto Tracing is broader and intended to foster communication between analysts and decision makers. The enumeration of many, possibly distinct, context and expectation sets provides an opportunity for broader consideration than perturbations of a baseline optimization formulation through traditional methods. Ongoing research is developing a more rigorous mathematical treatment of NPT, as well as further case study applications for illustration of the strengths and weaknesses of the approach.

REFERENCES

[1] A.M. Ross, and D.E. Hastings. "The Tradespace Exploration Paradigm," 15th *INCOSE International Symposium*, Rochester, NY, June 2005.

[2] R.L. Keeney, *Value-Focused Thinking: A Path to Creative Decisionmaking,* Cambridge, MA: Harvard University Press, 1992.

[3] R.L. Keeney and H. Raiffa, *Decisions with Multiple Objectives-- Preferences and Value Tradeoffs*, 2nd ed., Cambridge, UK: Cambridge University Press, 1993.

[4] A.M. Ross, "Multi-Attribute Tradespace Exploration with Concurrent Design as a Value-centric Framework for Space System Architecture and Design,". Cambridge, MA: MIT, SM in Aeronautics and Astronautics and Technology & Policy, pp. 258, June 2003.

[5] A.M. Ross, D.H. Rhodes, and D.E. Hastings, "Defining Changeability: Reconciling Flexibility, Adaptability, Scalability, Modifiability, and Robustness for Maintaining Lifecycle Value," *Systems Engineering*, vol. 11, no. 3, pp. 246-262, Fall 2008.

[6] A.M. Ross and D.H. Rhodes, "Architecting Systems for Value Robustness: Research Motivations and Progress," *2nd Annual IEEE Systems Conference*, Montreal, Canada, April 2008.

[7] A.M. Ross, "Managing Unarticulated Value: Changeability in Multi-Attribute Tradespace Exploration," Cambridge, MA: MIT, PhD in Engineering System, pp. 361, June 2006.

[8] O. de Weck, "Multiobjective Optimization: History and Promise," *Third China-Japan-Korea Joint Symposium on Optimization of Structural and Mechanical Systems*, Kanazawa, Japan, October-November 2004.

[9] A.M. Ross and D.H. Rhodes, "Using Natural Value-centric Time Scales for Conceptualizing System Timelines through Epoch-Era Analysis," 18th *INCOSE International Symposium*, Utrecht, the Netherlands, June 2008.

[10] R. Smaling, "System Architecture Analysis and Selection under Uncertainty," Cambridge, MA: MIT, PhD in Engineering Systems, pp. 48-50, June 2005.

[11] T.J. Spaulding, "Tools for Evolutionary Acquisition: a Study of Multi-Attribute Tradespace Exploration Applied to the Space Based Radar," Cambridge, MA: MIT, SM in Aeronautics and Astronautics, June 2003.

[12] A.M. Ross, H. McManus, et al., "Responsive Systems Comparison Method: Case Study in Assessing Future Designs in the Presence of Change," *AIAA Space 2008*, San Diego, CA, September 2008.

[13] A.M. Ross, H. McManus, D.H. Rhodes, D.E. Hastings, and A. Long, "Responsive Systems Comparison Method: Dynamic Insights into Designing a Satellite Radar System," *AIAA Space 2009*, Pasadena, CA, September 2009, unpublished.