



## SEARI Short Course Series

Course: PI.27s Value-driven Tradespace Exploration for System Design

Lecture: Lecture 3: Introduction to Multi-Attribute Decision Making

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This course was taught at PI.27s as a part of the MIT Professional Education Short Programs in July 2010 in Cambridge, MA. The lectures are provided to satisfy demand for learning more about Multi-Attribute Tradespace Exploration, Epoch-Era Analysis, and related SEARI-generated methods. The course is intended for self-study only. The materials are provided without instructor support, exercises or “course notebook” contents. Do not separate this cover sheet from the accompanying lecture pages. The copyright of the short course is retained by the Massachusetts Institute of Technology. Reproduction, reuse, and distribution of the course materials are not permitted without permission.



**Systems Engineering Advancement Research Initiative**

## ***[PI.27s] Value-Driven Tradespace Exploration for System Design***

### **Lecture 3**

### **Introduction to Multi-Attribute Decision Making**

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Massachusetts Institute of Technology



# Outline

- Introduction to decision theory
- Stakeholders-attributes-utilities
- Simple multi-attribute methods
- Example: personal multi-attribute decisions (cars, vacations)

# Making Decisions

At a fundamental level, design is about constrained “choice”

- Designers: choice of tools, concepts, colleagues, work hours, technology, etc.
- Users: CONOPS, reflected needs, anticipated needs, risk aversion and gaming, etc.
- Customers: benefit at cost, whose benefit, time value of money, etc.

Question: What makes a good “choice”?

Answer: Perceived benefit?

- Complicating factors: time, uncertainty, relative vs. absolute costs and benefits, distribution of costs and benefits...

How can design be improved through a “choice” point of view?

# An Example Decision

## Objectives

Buy a good house

## Goals

Low price/sq ft  
Be near a supermarket  
Good schools nearby  
Large square footage  
Good prospect for property value increasing  
Short commute to work  
Low utilities cost  
...

## Possibilities

1124 Elm St,  
Newton, MA  
326 Harvard St,  
Cambridge, MA  
477 Main St,  
Cambridge, MA  
37 Spruce Ter,  
Somerville, MA  
63 Lloyd Pl,  
Brookline, MA  
455 Patterson Blvd,  
Quincy, MA  
75 Lowell Hwy,  
Jamaica Plain, MA  
...

With so many possibilities, how to make sure choice is a “good” one?

# Decision Analysis Defined

**Decision analysis** “seeks to apply logical, mathematical, and scientific procedures to... decision problems of [consequence] that are characterized by...” uniqueness, importance, uncertainty, long run implications, and complex preferences

from Matheson, J.E., and Howard, R.A., “An Introduction to Decision Analysis” in *Readings in Decision Analysis*, Menlo Park: Stanford Research Institute, 1968

- Aimed at helping with “choice under uncertainty”
- Informs inter-temporal choice and choice within a set of competing decision makers

Most importantly, decision analysis structures the decision process to clarify the relationship among objectives, goals, possibilities, and *what makes a “good” choice*

# Types of Decision Theories

- Normative versus Prescriptive
  - How *should* people make decisions? (Given reality...)
    - Utility theory
    - Subjective Probability Theory
    - Modifications of various theories
- Descriptive
  - How *do* people *actually* make decisions?
    - Prospect Theory
    - Econometrics
- Decide to use prescriptive to “aid”
  - Decision analysis
  - Decision “support tools”

For any decision theory used, be aware of embedded assumptions about decision makers and their expected behavior

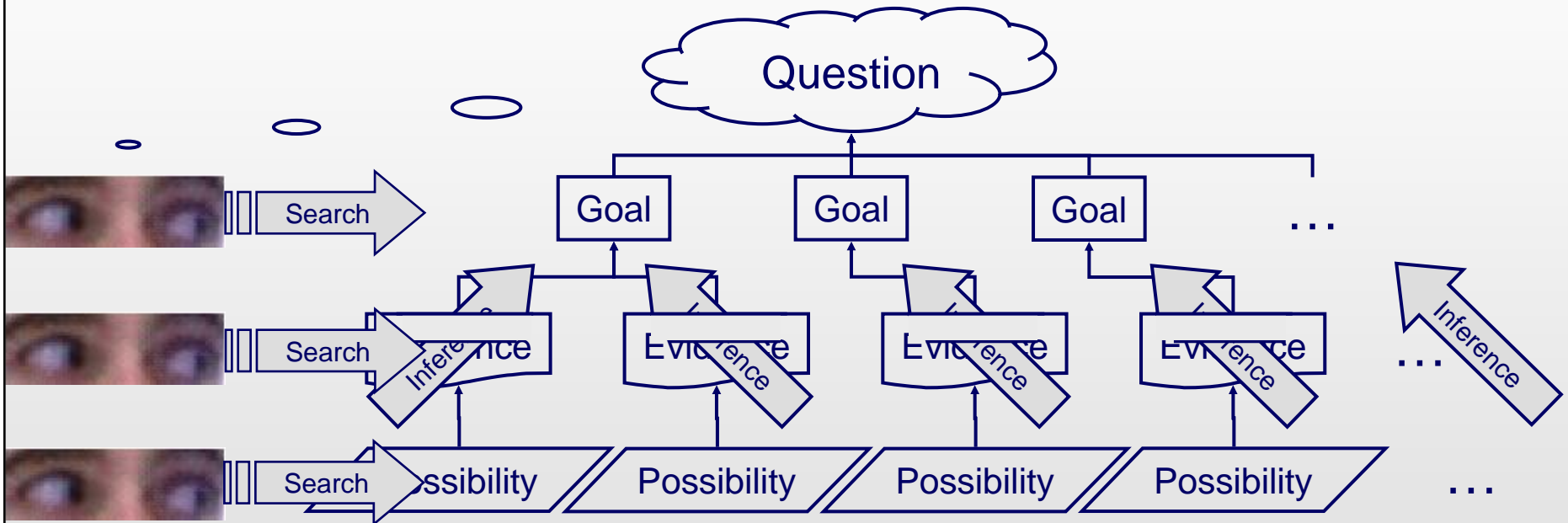
# Elements of a Decision

- Set **objectives**
- Generate **goals** for meeting objectives → selection criteria
- Generate alternatives → **possibilities** for meeting goals
- Evaluate alternatives in terms of criteria
- Rank alternatives based on criteria scores
- Select alternative(s) based on selection rule



# Thinking and Deciding: Search-Inference Framework

## A Model of Thinking: Search-Inference



“For any choice there must be purposes or goals, and goals can be added to or removed from the list. ...search for goals is *always* possible”\*

**People *will* change their minds**

\* Baron, Jonathan. *Thinking and Deciding: “Chapter 1—What is Thinking?”* Cambridge: Cambridge University Press, 2000. p7-9.  
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# Thinking and Deciding: Search-Inference Framework

## A Model of Thinking: Search-Inference

1. Given **goal(s)**, *search* for **possibilities**.
2. Make *inferences* on **evidence** to determine to what extent possibility achieves goal(s).

**Question:** Dilemma to be solved (Mission Objective)

**Goals:** Criteria used to evaluate possibilities (Attributes)

**Possibilities:** Possible answers to a question (System Designs)

**Evidence:** Belief or potential belief that helps to determine extent to which a possibility achieves some goal (System Analysis)

Search can be on goals, possibilities, and evidence—anything that may answer the question...

“For any choice there must be purposes or goals, and goals can be added to or removed from the list. ...search for goals is *always* possible”\*

**People *will* change their minds**

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## Value-focus vs. Alternatives-focus

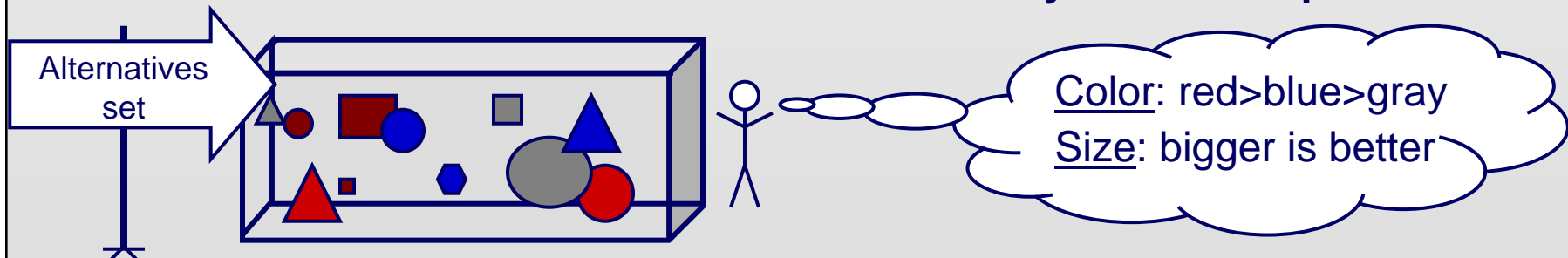
- Value-focused thinking\* starts with the rationale (i.e., value creation)
- Alternatives-focused thinking starts with the answers (i.e., solutions, designs, alternatives)
- Using alternative-focused thinking
  - May result in only small number of (possibly inappropriate, or subpar) solutions considered
  - More quickly reduces ambiguity in the problem by quickly getting to the concrete evaluation and specification part of design
  - May result in using scarce resources developing solutions that need to be justified (e.g., “sold” through altering expectations)
- Using value-focused thinking
  - Allows for “re-casting” of a “problem” into an “opportunity”
  - Increases likelihood of solution performing well in value
  - Aligns scarce resources on the proper questions
  - Allows for consideration of new solutions (helps to break anchoring)

**Value-focused thinking, or value-driven design, in general, will take more effort, but is far more likely to deliver a superior result**

\*Keeney, R. *Value-Focused Thinking—A Path to Creative Decision-making*, Cambridge: Harvard University Press, 1992

# So Design Is About Choice?

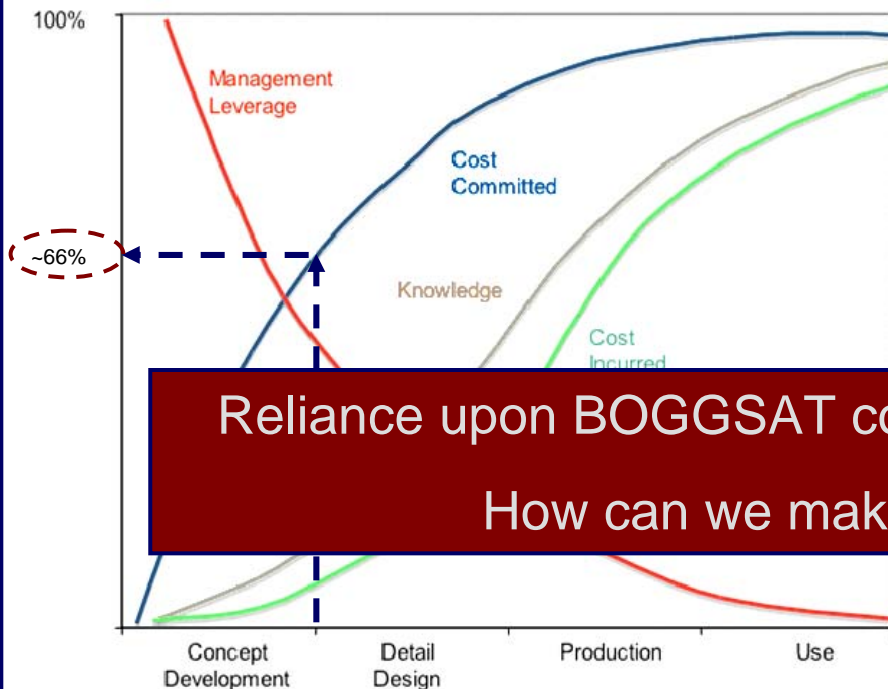
- “Choice” making a selection from a set of alternatives
- Classical decision theory concerns this problem
- Design encompasses a special class of decision problems: “wicked”
  - Open set of alternatives (infinite(?) possibilities)
  - Multi-criteria selection rule (multiple goals)
- Not a well-defined, theoretically solved problem...



**Actually “Design” is about creating “good” alternatives**

# The Scope of Upfront Decisions

Conceptual Design is a high leverage phase in system development

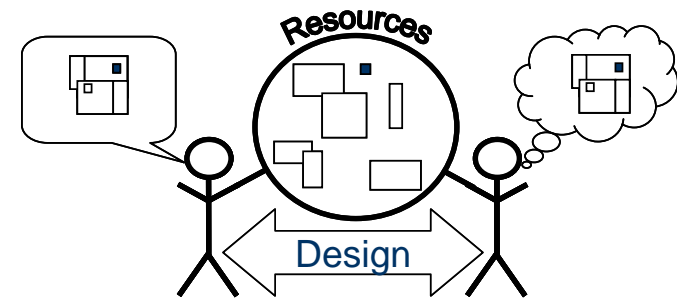


Reliance upon BOGGSAT could have large consequences  
How can we make better decisions?

After Fabrycky and Blanchard 1991

## Key Phase Activities

Needs Captured



Resources Scoped



Concept(s) Selected

# Three Keys to Good Upfront Decisions

- Structured program selection process
  - Choosing the programs that are right for the organization's stakeholders
- Systems engineering
  - Determining stakeholder needs, generating concept of operations, and deriving requirements
- Conceptual design practices
  - Finding the right form to maximize stakeholder value over the product (or product family) lifetime

**“Good” system decisions must both answer the *right questions* as well as answer the *questions right***

What is involved in getting the right questions?  
Decision Maker defined objectives

What is involved in getting the questions right?  
Rigorous decision and analytic methods

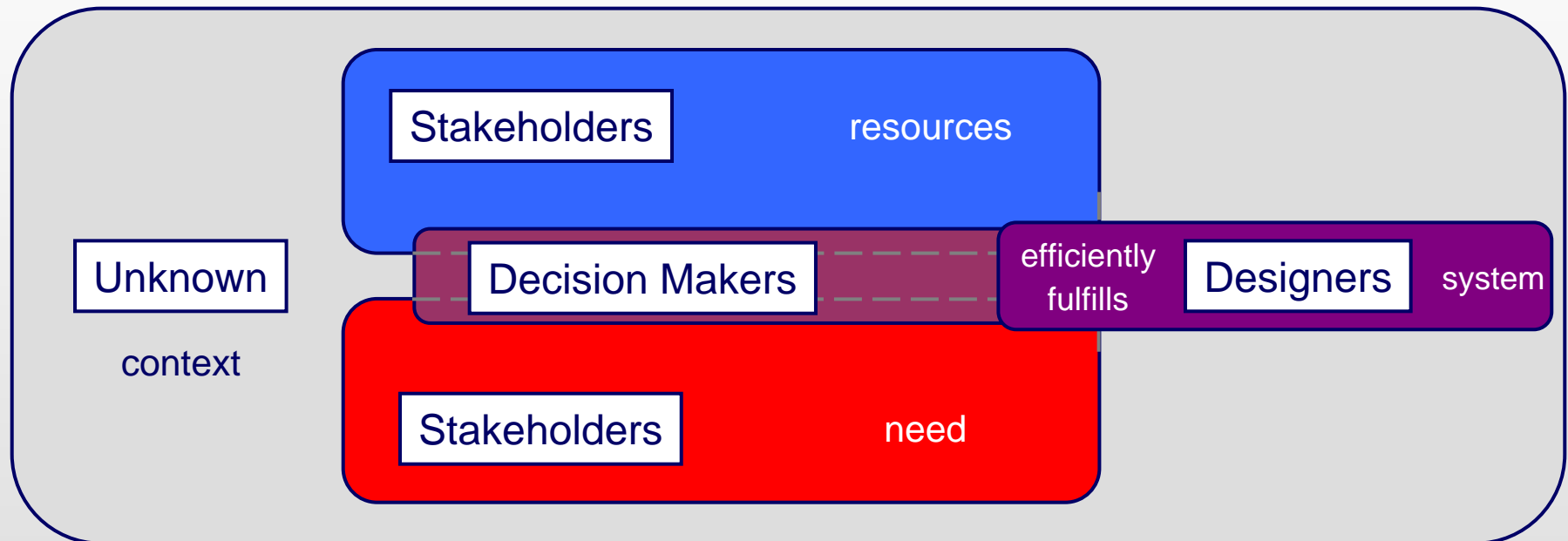


## Understanding the “Mission”

Identifying the “Value” Proposition  
(a.k.a. answering the *right question*)

# Determining “Value” Proposition

Create a system that fulfills some need while efficiently utilizing resources within some context



There may be many stakeholders, but...

Decision makers are a subset that have significant influence/control over the driving need and/or resources

... so focus on satisfying decision makers



# Framework for Identifying Key Decision Makers

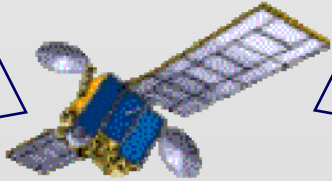
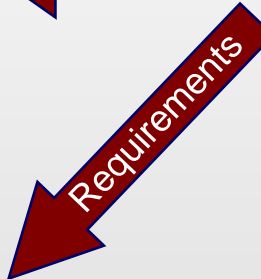
Level 0



Level 1



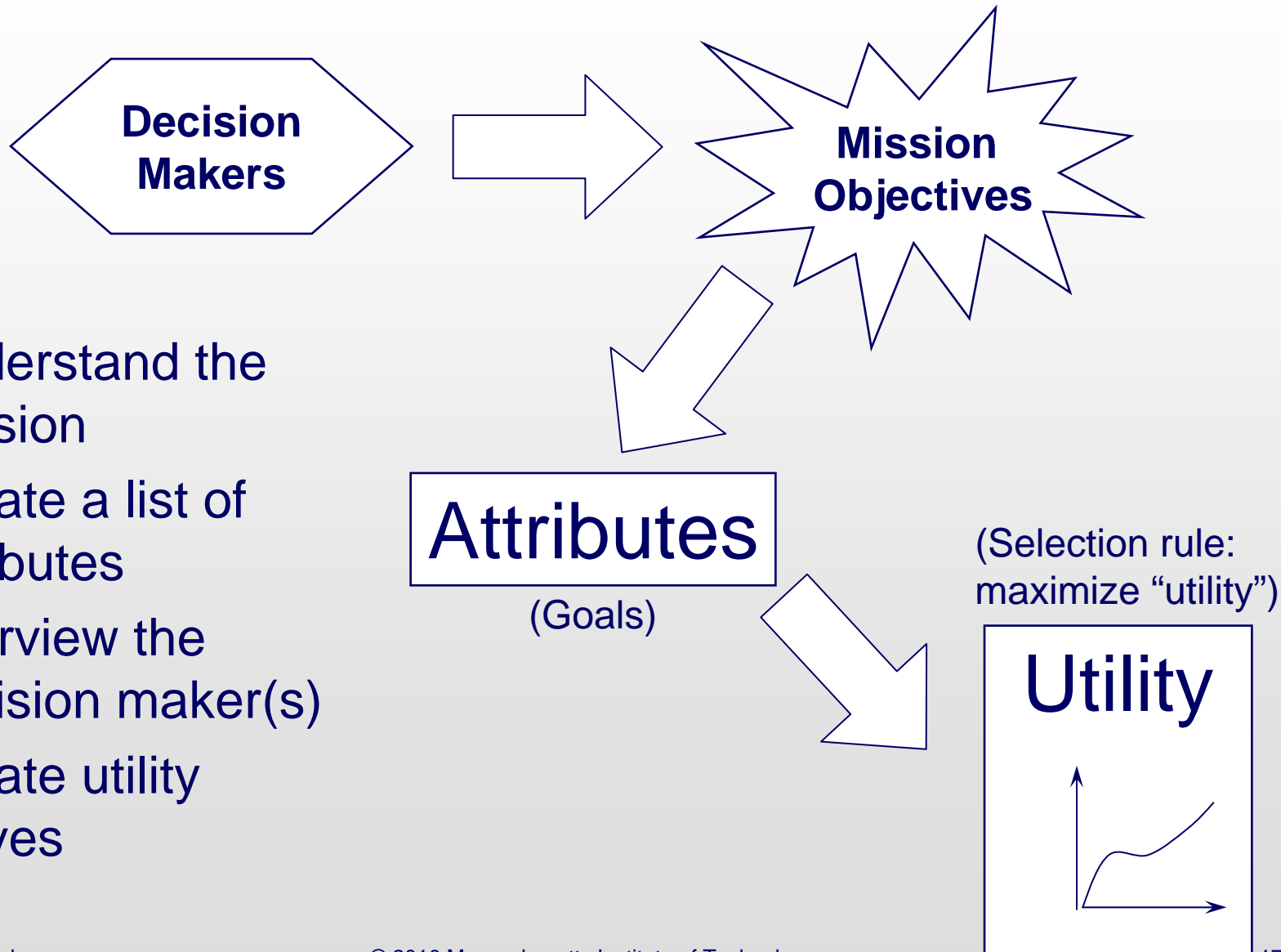
Level 2



## Definition of Levels

- Level 2 – Close connection to System
- Level 1 – Distant connection to System
- Level 0 – Little or no connection to System

# Define the Mission



- Understand the mission
- Create a list of attributes
- Interview the decision maker(s)
- Create utility curves

# Develop the “Mission” Statement

- Example from X-TOS

“design a conceptual space-based space system to characterize the upper atmosphere, with specific emphasis on the thermosphere and ionosphere. Building upon lessons learned from A-TOS and B-TOS, develop an architecture for the space system by March 22, 2002; building upon lessons learned from C-TOS, complete a preliminary design of this architecture by May 15th, and link this preliminary design back to the process used for the architectural study. Learn about engineering design process and space systems.”

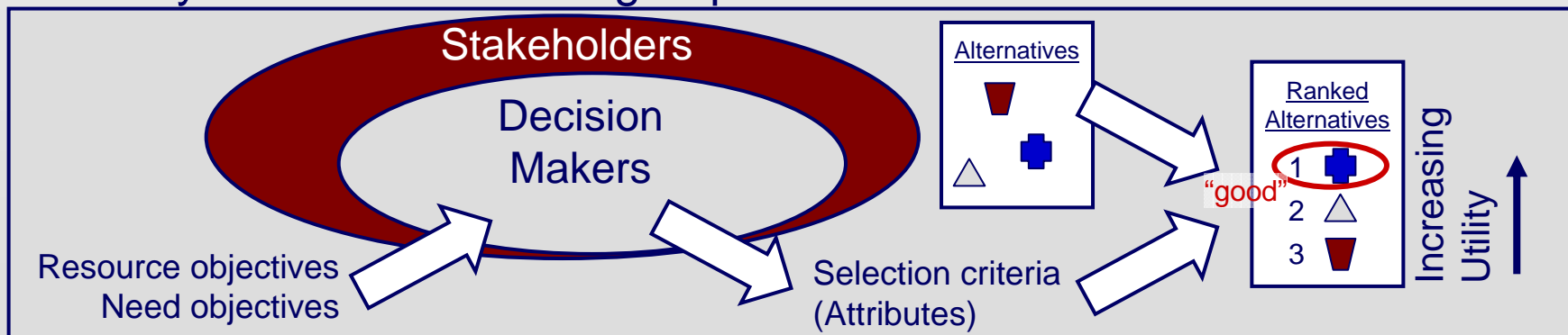
- Contains the following:

- Expectation and rough objective(s) of decision makers
- Imposed constraint(s)
- Possibly scoping decision(s)

A mission statement is often an encapsulation of an *implied* value proposition

# Stakeholders-Attributes-Utilities

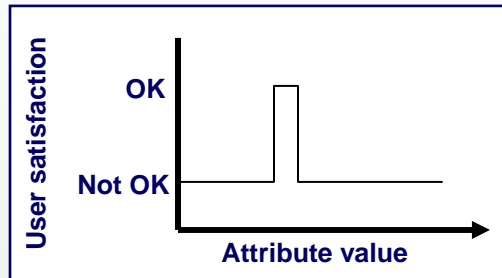
- In order to ensure a successful mission, the implied value proposition must be fulfilled
- Each system **stakeholder** has a value proposition—what they want to “get out” of the mission
- Decision makers are stakeholders with influence over the mission objectives for needs and/or resources
- Meeting the objectives for each decision maker can be assessed in terms of “**attributes**”
- An alternative that scores well in a set of attributes gives a decision maker value, or “**utility**”
- The goal for the selection of a good alternative is to maximize the utility for individuals and groups



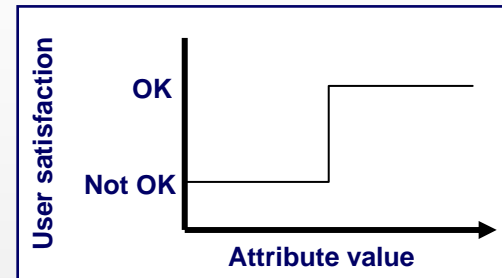
# Defining Attributes

- Decision criteria used by decision makers for selecting an alternative from among a set of alternatives
- Should obey, to the extent possible, perceived independence and other rules
- Set of 3-7 attributes “best” per individual DM
- For each attribute, must define:
  - Units: “natural” or “artificial”
  - Range: from lowest acceptable value, to highest meaningful value
- In practice, iteration may be necessary

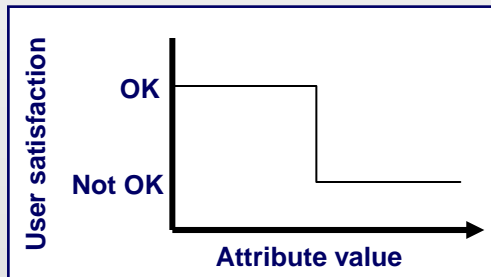
# Requirements: Binary Constraint on Attributes



a) Required value (range)



b) Must exceed

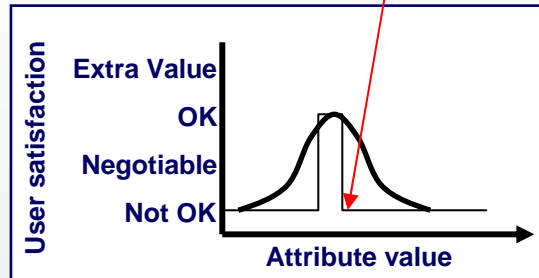


c) Must not exceed

If needs really are binary,  
model as constraints, not attributes

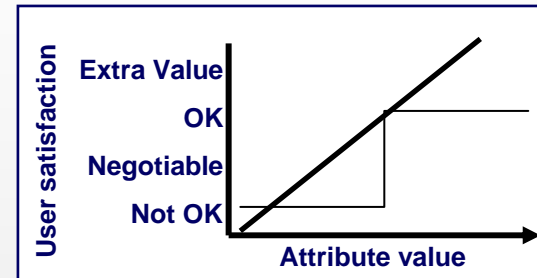
# Real Needs May Not be Captured

Is this a very bad spot?



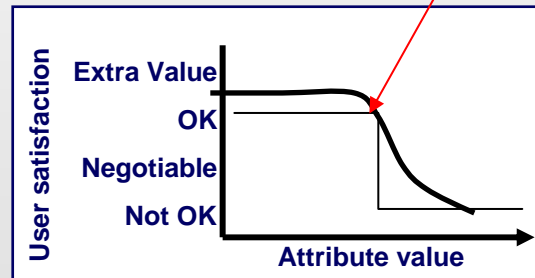
a) Soft target

Heroic effort to “make requirement,”  
or steady Improvement best?



b) More is better

Is this a good spot?

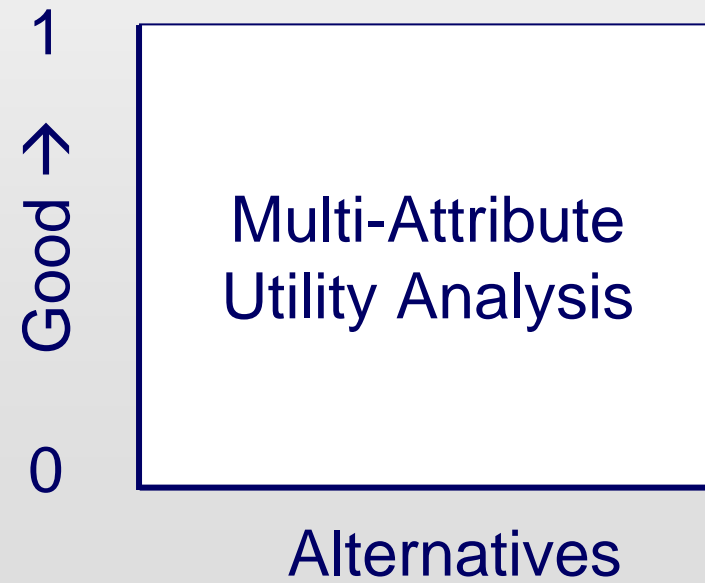
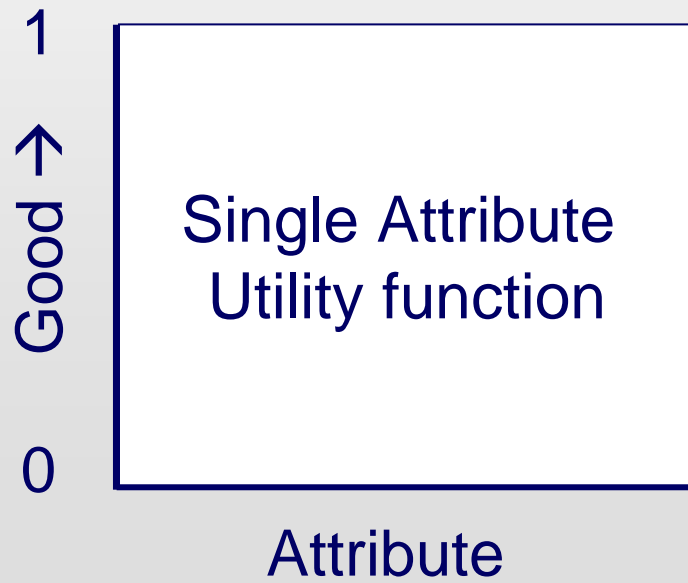


c) Soft threshold

Premature requirements definition may mask true objectives and goals

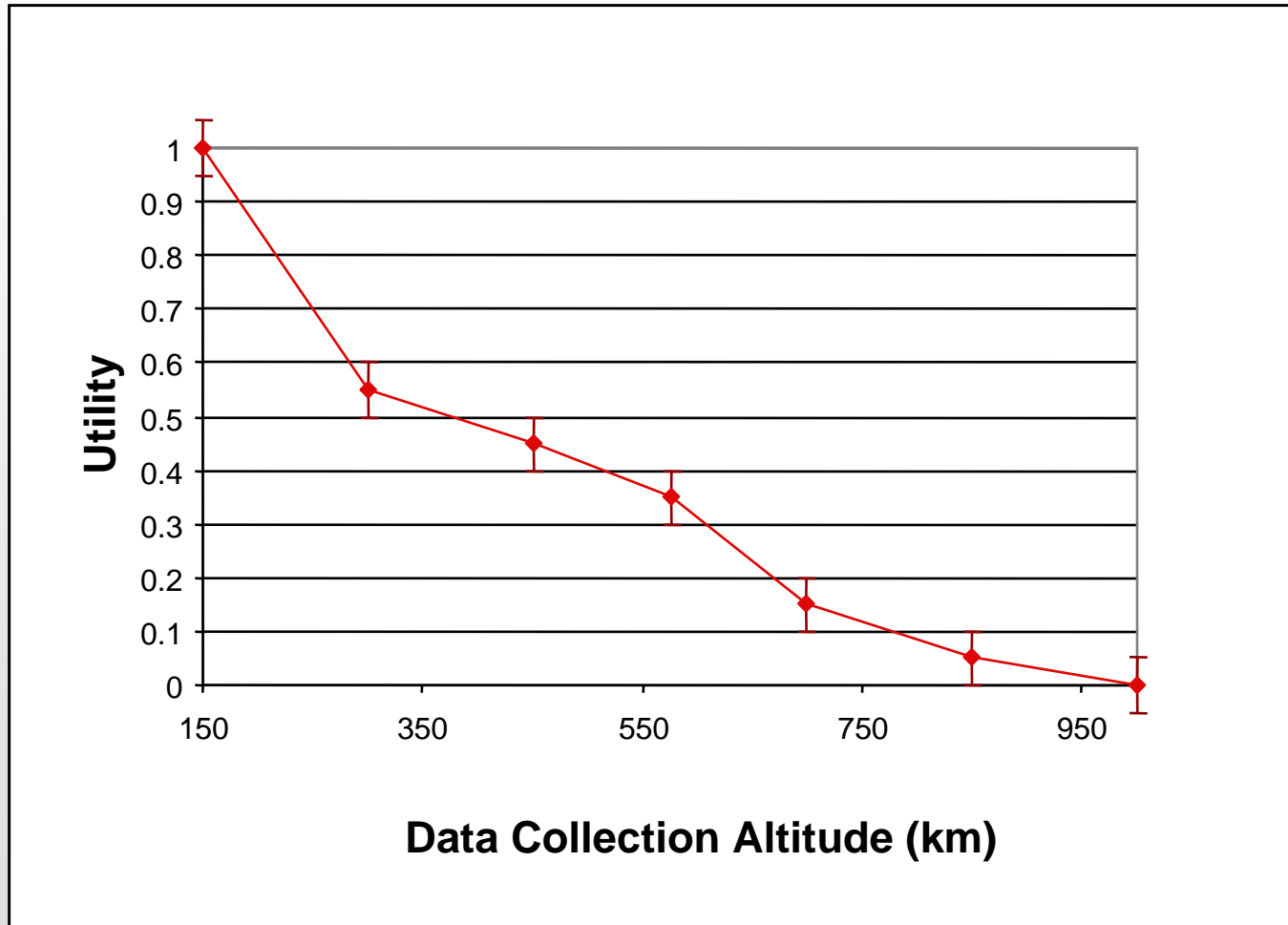
# Utilities as Selection Criteria

- “What the attributes are WORTH to the decision makers”
- Single Attribute utility maps attribute to utility
- Multi-attribute utility maps multiple attributes (as expressed by an alternative) to utility



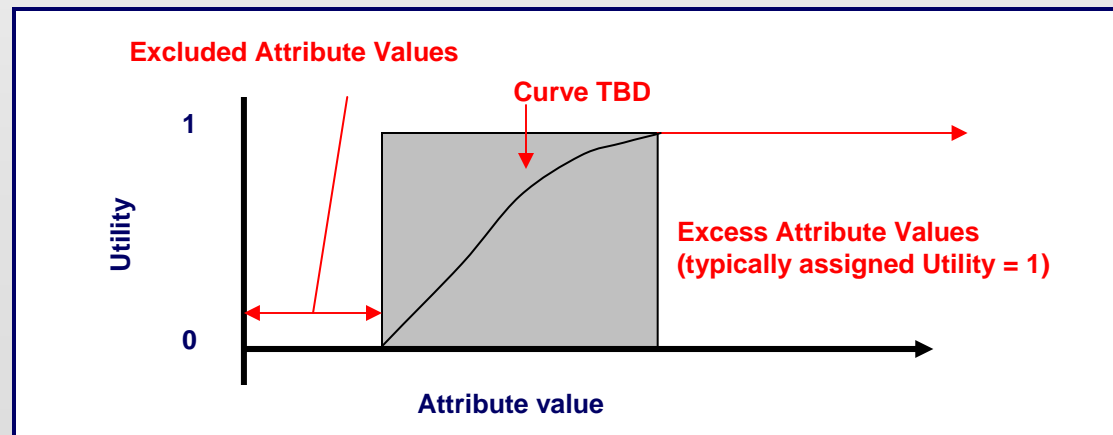


# Example Single Attribute Utility



# Single Attribute Utility

- Postulate a dimensionless metric that is a function of attribute  $X$ :  $U_i = U_i(X)$
- Set  $U_i = 0$  at the least desirable, *but still acceptable* value of  $X$
- Set  $U_i = 1$  at the highest (most desirable) value of  $X$
- $U_i$ , the “single attribute utility,” can be used to express the relative desirability of values of  $X$



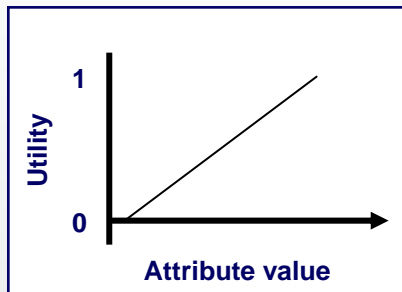
Source: “Utility Theory” lecture given by A. Ross for 16.892/ESD.353 Space Systems Architecture and Design, Oct. 2004

# Working with Utilities

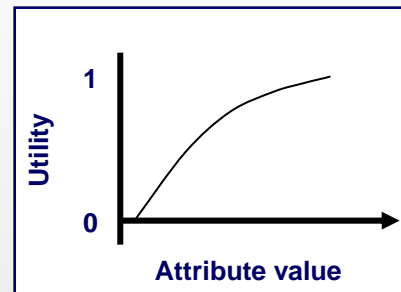
- Useful as selection criteria: contains complete ordering preferences
- Superior to pair-wise or other
- Determination methods:
  - Sketching
  - Analogy
  - Interviewing (more in lecture 5)

# Determining Single Attribute Utility

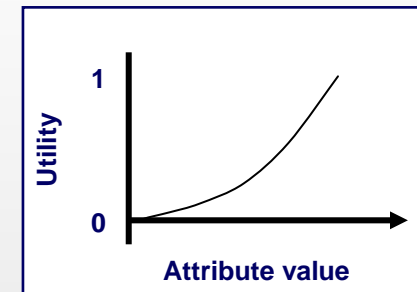
## Sketching - imprecise but easy



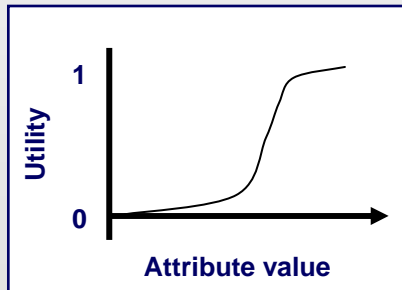
a) linear



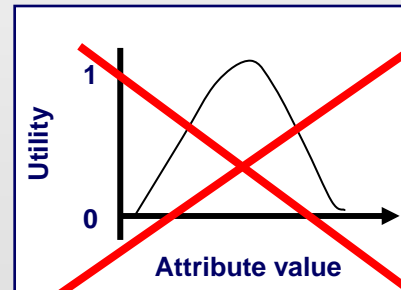
b) diminishing returns



c) increasing returns



d) threshold



e) non-monotonic

Not a permissible Utility  
Need to recast attribute definition

- Each curve shows “more is better”
- A curve could also be reversed (i.e., “less is better”)

**Beware: sketches are somewhat arbitrary; their use may result in “bad” decisions**

# Aggregating Utility for Single Decision Maker

- Single attribute may not be enough; multiple attributes must be aggregated for use in ranking
- Benefit:
  - Simplify analysis
  - Simplify communication
- Drawback:
  - Obscure trades
  - Misrepresent actual preferences
  - Impose preferences and biases
- Examples to follow

# Example Utility Aggregation Functions

- Combine multiple single-attribute utilities  $U_i$  into a single metric  $U$
- May not be possible
- May be simple

- Weighted sum  $U = \sum_{i=1}^n k_i U_i$

- Multiplicative function  $U = \prod_{i=1}^n U_i$

- Inverse multiplicative function  $1 - U = \prod_{i=1}^n (1 - U_i)$

- Generalized form - Keeney-Raiffa function

$$KU + 1 = \prod_{i=1}^n (Kk_i U_i + 1)$$

# Aggregating Utility Across Multiple Decision Makers

- Diversity across stakeholders
  - No absolute scale, so necessarily cannot compare numbers (no “anchor”)
  - Using utilities to rank options
  - Comparing rank as “absolute” measure
  - But what about “relative” weight of stakeholders?
- More discussion in lecture 5

Separate decision maker utilities *should* be kept separate; any combining of decision maker utilities *will* introduce assumptions and bias

# Simple Multi-Attribute Methods

- Lexicographic
  - Rank attributes
  - Score alternatives with natural units; (normalize scores)
  - Alternative with highest score in most important attribute is selected; if tied, tie-break with second most important attribute score, etc.
- Pugh
  - Choose baseline alternative
  - Determine comparison of each alternative's criteria to baseline: +/S/-
  - Sum +/S/- for each alternative; clear best alternative ranked first, etc.
- QFD
  - Rank attributes
  - Score alternatives with 1,3,9;
  - Alternative with highest score summed across attributes is selected
- Modified decision matrix
  - Rank attributes
  - Score alternatives with natural units; normalize scores
  - Alternative with highest weighted sum across normalized attribute scores is selected
- MAU
  - Elicit single attribute utility curves
  - Elicit multi-attribute utility attribute weights;
  - Score alternatives with single attribute utility curves
  - Alternative with highest multi-attribute utility is selected



# Lexicographic (Decision Matrix) Method

- Rank attributes
- Score alternatives with natural units; (normalize scores)
- Alternative with highest score in most important attribute is selected; if tied, tie-break with second most important attribute score, etc.

Importance:	0.3	0.2	0.4	0.1	
Criterion:	Accel	Comfort	Price	MPG	Rank
Volvo	8	7	60K	22	3
Porsche	6	10	90K	16	4
Kia	9.5	3	10K	29	1
Toyota	8.5	6	30K	34	2

## Natural units

Importance:	0.3	0.2	0.4	0.1	
Criterion:	Accel	Comfort	Price	MPG	Rank
Volvo	0.2	0.7	0.4	0.65	3
Porsche	0.4	1	0.1	0.47	4
Kia	0.05	0.3	0.9	0.85	1
Toyota	0.15	0.6	0.7	1	2

Normalized units: 0-1, 1 is better

## Alternatives Ranking

Rank	Car
1	Kia
2	Toyota
3	Volvo
4	Porsche

# Pugh (Controlled Convergence) Method

- Choose baseline alternative
- Determine comparison of each alternative's criteria to baseline: +/S/-
- Sum +/S/- for each alternative; clear best alternative ranked first, etc.

Weight:	0.3	0.2	0.4	0.1	
Criterion:	Accel	Comfort	Price	MPG	
Volvo	8	7	60K	22	
Porsche	6	10	90K	16	
Kia	9.5	3	10K	29	
Toyota	8.5	6	30K	34	

## Natural units

Criterion\Concept	Volvo (base)	Porsche	Kia	Toyota
Accel	<b>S</b>	<b>+</b>	<b>-</b>	<b>S</b>
Comfort	<b>S</b>	<b>+</b>	<b>-</b>	<b>S</b>
Price	<b>S</b>	<b>-</b>	<b>+</b>	<b>+</b>
MPG	<b>S</b>	<b>-</b>	<b>+</b>	<b>+</b>
$\Sigma+$	0	2	2	2
$\Sigma-$	0	2	2	0
$\Sigma S$	4	0	0	2

## Alternatives Ranking

Rank	Car
<b>1</b>	<b>Toyota</b>
<b>2</b>	<b>Volvo</b>
<b>2</b>	<b>Porsche</b>
<b>2</b>	<b>Kia</b>

Pugh, S., *Total Design: Integrated Methods for Successful Product Engineering*, Addison-Wesley: New York, 1991. 33

# QFD Method

- Rank attributes
- Score alternatives with 1,3,9;
- Alternative with highest score summed across attributes is selected

Weight:	0.3	0.2	0.4	0.1	
Criterion:	Accel	Comfort	Price	MPG	
Volvo	8	7	60K	22	
Porsche	6	10	90K	16	
Kia	9.5	3	10K	29	
Toyota	8.5	6	30K	34	

## Natural units

Weight:	0.3	0.2	0.4	0.1		
Criterion:	Accel	Comfort	Price	MPG	Sum	Wt Sum
Volvo	3	3	3	3	12	3
Porsche	9	9	1	1	20	5.9
Kia	1	1	9	9	20	5
Toyota	3	3	3	9	18	3.6

Qualitative units: 1=bad, 3=okay, 9=great

## Alternatives Ranking

Rank	Car
1	Porsche
2	Kia
3	Toyota
4	Volvo

# Modified Decision Matrix Method

- Rank attributes
- Score alternatives with natural units; normalize scores
- Alternative with highest weighted sum across normalized attribute scores is selected

Weight:	0.3	0.2	0.4	0.1	
Criterion:	Accel	Comfort	Price	MPG	Score
Volvo	8	7	60K	22	$\text{Sum}(W_j * X_{1j})$
Porsche	6	10	90K	16	$\text{Sum}(W_j * X_{2j})$
Kia	9.5	3	10K	29	...
Toyota	8.5	6	30K	34	$\text{Sum}(W_j * X_{Mj})$

Natural units

Weight:	0.3	0.2	0.4	0.1		
Criterion:	Accel	Comfort	Price	MPG	Score	Rank
Volvo	0.2	0.7	0.4	0.65	0.425	3
Porsche	0.4	1	0.1	0.47	0.407	4
Kia	0.05	0.3	0.9	0.85	0.52	2
Toyota	0.15	0.6	0.7	1	0.545	1

Normalized units: 0-1, 1 is better

Alternatives Ranking

Rank	Car
1	Toyota
2	Kia
3	Volvo
4	Porsche

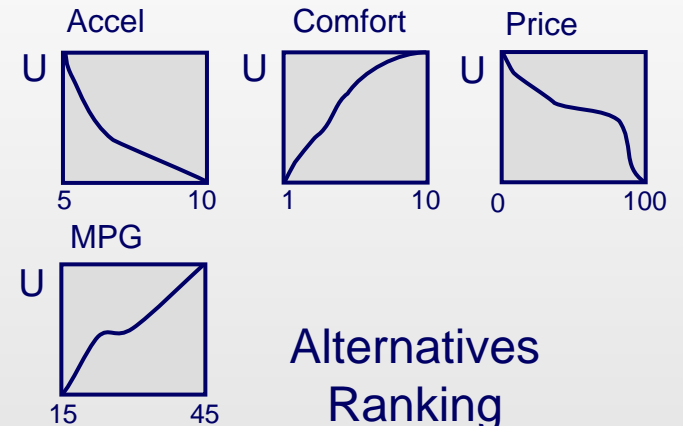
# MAU Method

- Elicit single attribute utility curves
- Elicit multi-attribute utility attribute weights;
- Score alternatives with single attribute utility curves
- Alternative with highest multi-attribute utility is selected

$$U = \sum_{i=1}^n k_i U_i$$

Weight:	0.3	0.2	0.4	0.1	
Criterion:	Accel	Comfort	Price	MPG	
Volvo	8	7	60K	22	
Porsche	6	10	90K	16	
Kia	9.5	3	10K	29	
Toyota	8.5	6	30K	34	

Natural units



Alternatives Ranking

Weight:	0.3	0.2	0.4	0.1	K=1	
Criterion:	Accel	Comfort	Price	MPG	Score	Rank
Volvo	0.25	0.85	0.6	0.4	0.525	2
Porsche	0.7	1	0.1	0.05	0.455	4
Kia	0.05	0.35	0.9	0.5	0.495	3
Toyota	0.15	0.75	0.7	0.7	0.545	1

Rank	Car
1	Toyota
2	Volvo
3	Kia
4	Porsche

Utility units: 0-1, 1 is better

# Comparison of MA methods

Answering the question right...

... requires understanding the limits of the method used

Rank	Car	Rank	Car	Rank	Car	Rank	Car	Rank	Car
1	Kia	1	Toyota	1	Porsche	1	Toyota	1	Toyota
2	Toyota	2	Volvo	2	Kia	2	Kia	2	Volvo
3	Volvo	2	Porsche	3	Toyota	3	Volvo	3	Kia
4	Porsche	2	Kia	4	Volvo	4	Porsche	4	Porsche

Lexicographic

Pugh

QFD

Modified DM

MAU

More effort to rank



MAU is *axiomatically* based and helps for answering the right question as well as answering the question right (more in Lecture 4)

Among only four alternatives, each method resulted in a *different* ranking!

# Buying a Car



VS.



<http://www.carpictures.com/>

# Example Application 1: Buying a Car

## Example decision makers:

- User
- Consumer
- Manufacturer
- Dealer
- Government

## Example attributes:

- Gas mileage
- Aesthetic appeal
- Sound system
- Acceleration
- Handling
- Sex appeal
- Price
- Maintenance cost
- Reliability
- Engine sound
- Passenger capacity
- Uniqueness
- Having gull-wing doors
- Availability
- Emissions
- Safety
- Premium (price)
- Unit Cost